The Middle Opinion. USA 2020.

Chapter 14 (pre-election draft)[1]

Optimal social choice, value functions:

social design, ethics and the amount of value[2]

Social ethics addresses the question 'what should be done in society?'.
Kolm (1998, p. 3)

'Pigou thought that welfare economics was a potent instrument for the bettering of human life.'
Suzumura (2002, p. 26)

'These choices along these various dimensions are indeed relevant to us and we will be seeking over the next period to find these balances.'
A social design practitioner (see the case study in this chapter)

Ethics is a complex subject and here we focus on a specific ethical criterion, the utilitarian social welfare function. The ideas are relevant to other values besides welfare and the maximisation of total welfare may under certain circumstances be associated with the minimisation of inequality. The notion of value in this chapter is that an object can have a certain amount of value for an individual. Limitations on social value are noted. There are tensions between competing options. The provision of more than one option allows some relaxation of these limitations and tensions. If the option space is continuous then the social value function can take a variety of specific forms. The notion of value-generating power is introduced. Given certain assumptions, the mean social value is a maximum at the mean ideal. Sub-optimal social value can arise as a result of the following factors: a sub-maximal value of the best option; population variation in ideals; the distance of the provided option from the best option; and sensitivity to deviation from the ideal. Practical social design requires attention to a variety of design dimensions and knowledge about people's values regarding these dimensions. This knowledge may not be known in advance and so the design process can be usefully informed by the identification of design dimensions and the obtaining of evidence about people's values regarding these dimensions. An application of these ideas to educational design is described.

6.1 Ethics and the utilitarian social welfare function

According to Kolm (1998, p.3), social ethics addresses the question 'what should be done in society?'. The topic of justice constitutes a very large part of social ethics

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although other virtues are also important. Kolm distinguishes between macrojustice and microjustice. For the former, Kolm proposes ‘a combination of the three rationales of rights and duties about capacities: process-freedom, partial income equalization by efficient means, and the satisfaction of basic needs and the alleviation of deep suffering’. Sen (1991, pp. ix, 21-22, 150) argues that ‘a common characteristic of virtually all the approaches to the ethics of social arrangements that have stood the test of time is to want equality of something – something that has an important place in the particular theory’. For example even libertarian thinkers such as Nozick who are perceived as being anti-egalitarian place importance on people having liberty and hence that equality of liberties is important. Sen’s own capability approach ‘has something to offer both to the evaluation of well-being and to the assessment of freedom’.

The proposals of Kolm and Sen reveal the complexity of the literature on ethics. In contrast the focus in this chapter is on the quite simple notion of a utilitarian social welfare function.

‘The utilitarian form is by far the most common and widely applied social welfare function in economics. Under a utilitarian rule, social states are ranked according to the linear sum of utilities.’
(Jehle and Reny, 2001, p. 255)

In its selective focus the chapter shall be ignoring a variety of issues. Thus despite its ethical importance, inequality will not be explicitly discussed. However, noting Cowell’s (1995, p. 21) list of measures of inequality (range, relative mean deviation, variance, coefficient of variation, Gini coefficient and log variance) and Rawls concern for the welfare of society’s worst-off (Jehle and Reny, 2001, p. 252), we shall be interested to see under what circumstances maximizing total utility also minimizes these inequality criteria.

Social welfare theory has a quite specific focus on the welfare consequences of the options for individuals, ‘welfarist-consequentialism’ (Suzumura, 2002, pp. 23-25). Both the welfare and the consequentialist part of this can be challenged. It can be argued as noted above that options have value for individuals quite apart from the value associated with welfare consequences – for example the values of individual liberty, social primary goods, resources, capabilities … conferment and realization of rights, etc. However much of the formalism of social welfare theory can be applied equally well to any type of value and so much of the discussion of this chapter may have relevance beyond issues of social welfare.

Next it might be argued that certain aspects of social value are not connected to the value for individuals. While some might argue that truth and beauty were subjective (and hence related to the values of individuals) others might say that they were absolute (and hence not related to the values of individuals).

Finally the utilitarian social welfare function assumes that objects have a certain amount of value. This is an issue which we now turn to.

6.2 The amount of value

Although the previous two chapters have discussed value they have not conceived of value as an amount. Instead they have conceived of value either in binary terms (an object either has value or it does not) or in ordinal terms or in terms of preference. Much of the literature regards preference as the primary concept and has reservations about the concept of an amount of value. Do people in reality place perceive an object as having an amount of value? People might be willing to say that they preferred A to B, but would they be willing to say that A had a certain amount of value and B had a certain amount of value? Even if people were willing to associate amounts of value with objects, how do we know that when two people say the give the same amount of value to an object that in fact they do place the same amount of value on it? This issue is referred to as the personal inter-comparability of values.

Despite these concerns the concept of an amount of value is very attractive and perhaps not as unrealistic as suggested in the previous paragraph. In order to define such a concept the social welfare literature introduces a number of additional assumptions to those made when discussing preferences. For example Jorgenson (1997) offers the following discussion, leading from the weaker assumptions underlying Arrow’s result to the stronger assumptions required for various classes of social welfare function. Arrow’s result makes rather weak assumptions about individual preferences – he assumes only ordinal non-comparability. Arrow’s result still holds if cardinal non-comparability is assumed. However if cardinal comparability is assumed then there exists a class of social orderings which can be represented by certain types of social welfare functions: cardinal unit comparability yields utilitarian social welfare functions; and cardinal full comparability yields a class of social welfare functions which are the sum of two components, an average of the individual welfare functions plus a measure of dispersion in individual welfare levels (Jorgenson, 1997, pp. 3, 63-67). Further assumptions yield other classes of welfare functions (Jorgenson, 1997, pp. 3, 67-72).

6.2.1 The individual and social value of objects and attributes

In this section we provide a formal treatment of the following ideas. The basic notion is that an individual regards an object as having a certain amount (or quantity) of value. The object has certain attributes and the value of the object for the individual depends on the value of the attributes for the individual. There is a set of individuals and the social value of the object for the set of individuals depends on the values which the individuals place on the object. A common assumption about these dependences is that they are additive. For example the utilitarian social welfare function assumes that individual values can be added to form the social value. Sometimes interest centres on total social value and sometimes on mean social value – the two measures order states in the same way when the number of individuals is constant.

Here we confine our attention to the utilitarian social welfare function. In equation (1) below, x is a social state, U is the social welfare function based on the vector $\underline{u}(x)$ of individual welfare functions $u_i(x)$ for individuals i=1, … N, and the 'welfare weights' w_i are constants with $\Sigma_{i=1}^{N} w_i=1$, $w_i \geq 0$.

$$U[\underline{u}(x)] = \Sigma_{i=1}^{N} w_i u_i(x) \qquad (1)$$

The set of social states may be discrete but here we make the assumption that the set of social states is a subset of a multi-dimensional real space. We say a utility function is separable if it can be expressed as the sum of functions, with each function dependent on just one dimension of the social state space. There are many situations where the utility functions are not separable. However here we focus on the simpler case of separable utility functions.

In the additive version of multi-attribute utility theory, the utility of an option is a weighted sum of the utilities of its attributes, the weights being the sensitivities of the dimensions. This has been found to be quite a robust model. (Borcherding et al., 1995; Diederich, 1995). Suppose the attributes are labelled j=1, …M. The social state $\underline{x}$ has co-ordinates $\{x_j\}$; and w_{ij} denotes the sensitivity attached to attribute j by individual i.

$$u_i(\underline{x}) = \Sigma_{j=1}^{M} w_{ij} u_i(x_j) \qquad \text{with } \Sigma_{j=1}^{M} w_{ij} = 1,\ w_{ij} \geq 0 \qquad (2)$$

Substituting this in equation (1) we can obtain two equivalent expressions for the social welfare function, either as the weighted sum of the individual utilities $u_i(\underline{x})$ or as the weighted sum of the social welfares U_j generated by each dimension j.

$$U[\underline{u}(\underline{x})] = \Sigma_{i=1}^{N} w_i\, u_i(\underline{x}) \qquad (3)$$

$$U[\underline{u}(\underline{x})] = \Sigma_{j=1}^{M} W_j\, U_j \qquad (4)$$

where $U_j = \Sigma_{i=1}^{N} a_{ij}\, u_i(x_j)$ and $W_j = \Sigma_{i=1}^{N} w_i\, w_{ij}$; $a_{ij} = (w_i w_{ij} / W_j)$; and $\Sigma_{i=1}^{N} a_{ij} = 1$.
This follows from $U[\underline{u}(\underline{x})] = \Sigma_{i=1}^{N} w_i \Sigma_{j=1}^{M} w_{ij} u_i(x_j) = \Sigma_{j=1}^{M} \Sigma_{i=1}^{N} w_i\, w_{ij}\, u_i(x_j)$.

6.2.2 Limitations on the value of a social design

Given a choice between social options, social design seeks to maximize social value but there are certain limitations on social value which need to be recognized. In particular mean value is limited because individuals have conflicting utility functions. There may be substantial conflict between two options with the same mean value.

The value v of a social outcome lies within a nest of intervals. Firstly it must lie in the interval V_1 between the minimum possible value and the maximum possible value. Next it must lie in the interval V_2 between the mean of individuals' minimum values and the mean of individuals' maximum values. Next it must lie in the interval V_3 between the mean of the option with the lowest value and the mean of the option with the highest value.

So reality falls short of perfection! The value can be expressed as the sum of three gaps or deficits from the maximum possible value. Let's start with perfection – the maximum possible value. The first problem is that even when an option is the one which is best for an individual, still the value may fall short of the maximum possible value. The second problem is that different individuals may have different ideal options. As a result even the best option may have less value than the mean value of individual best values. The third problem is that a given option may not be the best option.

$$v = v_{1max} + (v_{2max} - v_{1max}) + (v_{3max} - v_{2max}) + (v - v_{3max})$$

$$v = v_{1max} + g_{12} + g_{23} + g_{3}$$

Even when options have the same mean value there may be considerable conflict between the options. Comparing any two options A and B, we can identify three groups. In one group with n_1 individuals the group's means are $m_{A1}>m_{B1}$; in a second group with n_2 individuals the group's means are $m_{A2}<m_{B2}$; and in a third group with n_3 individuals the group's means are $m_{A3}=m_{B3}$. If the option selected is A then the first group experiences a total gain of $n_1(m_{A1}-m_{B1})$; the second group experiences a total loss of $n_2(m_{A2}-m_{B2})$; and the third group experiences indifference – all of these results in comparison with the value of option B. The total gain equals the total loss if and only if the two options have the same mean value. So even though two options have the same value there may be substantial losses and gains by different groups depending on the option chosen.

Finally, thinking about the different measures of inequality, we note a relationship between the criterion C of maximizing the minimum value and the criterion C' of minimizing the range of values. If, for every option, the maximum value is always achieved by some individual then an option which secures criterion C also secures criterion C'.

6.3 Social value functions on a continuous space

Now the ideas of the previous chapter are introduced, namely that the set of options may be a continuous or ordered space. With this assumption we now wish to consider in greater detail the form of the social welfare function. From equation (1) above, the form of the social welfare function depends on the form of the individual utility functions. Because the utility functions are separable it is sufficient for some purposes to consider just one dimension. First note that there are a number of results which apply to all types of utility functions. Secondly some utility values may not be systematically related to the ordering of the dimension. For functions that have a systematic relationship, following Gottfried and Weisman (1973, p. 1-21), the functions can be classified according to whether they are: unimodal (single-peaked) or multimodal (multi-peaked); symmetric about the peak(s) or not; convex or concave or a mixture of these two types; continuous or not (in the continuous case, a convex function has a decreasing derivative); their degree; one-variable or multi-variable; and simple or compound (that is, a function of variables which are themselves functions of variables).

6.3.1 Single-peaked functions

If the social welfare function is multi-peaked then local improvements may not move the situation towards the global optimum. The situation is simpler if the social welfare function is single-peaked. For this to happen it is sufficient that the individual utility functions are single-peaked with decreasing derivatives – see Theorem 6.1 below. First though a few preliminary results.

Consider just one dimension, x. Suppose that, for individual i, the utility on this dimension is a single-peaked function $u_i(x)$, attaining a maximum when $x=x_i$. We refer to x_i as the individual ideal (for that individual). So utility decreases with distance either side of the individual ideal. Consider now a set I of individuals. Let $a=\min\{x_i\}$ and $b=\max\{x_i\}$

Result 6.1

Suppose the individual utility functions are single-peaked. Then the criterion of maximizing the minimum welfare is achieved at a point x such that $v_a(x)=v_b(x)$ and x is in the closed interval [a,b].

Theorem 6.1

Suppose the individual utility functions are single-peaked. If the individual utility functions are also differentiable with a derived function which is decreasing, then the social welfare function is also single-peaked. Where the maximum social welfare occurs is referred to as the welfare ideal.

Proof

By assumption, the $u_i(x)$ are differentiable and single-peaked and the $u_i'(x)$ are decreasing functions of x. Using $U[\underline{u}(x)]=\Sigma_{i=1}^{N}w_iu_i(x)$, we differentiate with respect to x to obtain: $U'=\Sigma_{i=1}^{N}w_iu_i'(x)$. So U' is a decreasing function of x. For $x<a$ all the $u_i(x)$ are positive and for $x>b$ all the $u_i(x)$ are negative. So $U'(a-\varepsilon)>0>U'(b+\varepsilon)$ for all $\varepsilon>0$. So there exists x^* such that $U'(x^*)=0$ where $a\leq x^*\leq b$.

6.3.2 Distance functions (symmetric about the peak)

Single-peaked functions can be further classified into those which are symmetrical about the peak and those which are not. Symmetry is equivalent to value being a decreasing function of distance from the peak.

Result 6.2

Suppose the individual utility functions are single-peaked and symmetric and identical except for a lateral shift. Then the criterion C of maximizing the minimum welfare is achieved at a point x such that $v_a(x)=v_b(x)$ and $x=(a+b)/2$.

Result 6.3

If it is possible to supply more that one design and for individuals to choose their best design then the designs which optimize criterion C of maximizing the minimum welfare are placed at:

(a+d/4); and (a+3d/4) where two designs are provided;

(a+d/6); (a+3d/6); and (a+5d/6) where three designs are provided;

Etc.

The previous chapter obtained results relating power to the mean ideal and the median ideal. Attention was paid to the distances between the ideal and other points in the option space. An individual or group was thought to be better placed if the outcome was nearer the individual's ideal. In particular the case where the majority party was closer than the minority party to the outcome was discussed. These results have an additional interpretation if the value functions are identical distance functions, namely that being closer to one's ideal means experiencing greater welfare. For example one might define value-generating power as du_i/dx_i as the change in value for i dependent on the change in the ideal of i. Somewhat schematically we have $du_i/dx_i=(du_i/dx^*).(dx^*/dx_i)=k_i m_i w_i$, the product of a constant, the individual's value sensitivity to change in the option dimension and the power as defined in the previous section.

The two simplest types of symmetrical single-peaked function are the modulus function and the quadratic function and both types are commonly found as assumptions in the literature.

6.3.3 The modulus function

The modulus utility function specifies that the utility decreases linearly with distance from the ideal. In equation (5) below, the parameter c_i represents the utility ceiling and the parameter m_i represents the sensitivity of utility to changes in the social state.

$$u_i(x) = - m_i(|x-x_i| + c_i \qquad m_i \geq 0 \qquad (5a)$$

6.3.4 The quadratic function

It is possible to construct a quadratic Taylor approximation to the utility function near the ideal. Here we shall simply assume that the utility function *is* quadratic. In equation (5) below, the parameter c_i represents the utility ceiling and the parameter m_i represents the sensitivity of utility to changes in the social state.

$$u_i(x) = - m_i(x-x_i)^2 + c_i \qquad m_i \geq 0 \qquad (5b)$$

In this situation the utilitarian social welfare function in the form of equation (1) above can be re-expressed in terms of the ideals of the individuals in the population.

Theorem 6.2

Suppose that the set of social states is one-dimensional and that each individual has a quadratic utility function – as in equation (5) above. Suppose also that all individuals have the same sensitivity. Then the welfare ideal x is the weighted sum of the individual ideals. The social welfare of a situation depends on the population sensitivity m, the population weighted variation V, the deviation D of the situation from the welfare ideal and the welfare ceiling C.*

$$x^* = \Sigma_{i=1}^{N} w_i x_i \qquad (6)$$

$$U[\underline{u}(x)] = - m (V + D^2) + C \qquad (7)$$

$$V = V(\{x_i\}) = \Sigma_{i=1}^{N} [w_i(x_i-x^*)^2]$$
$$D = (x-x^*)$$

$C = \Sigma_{i=1}^{N} w_i c_i$

Corollary of theorem 6.2

For a given population, social welfare U is maximised and is at the level, -mV+C, when the social state is identical with the welfare ideal, x=x. If in addition all the population share the same ideal, $x_i=x^*$, and so V=0, then the social welfare is simply C. Taking the derivative of U with respect to x indicates the impact on social welfare of a unit change in design: dU/dx=-2mD.*

Proof

$U[\underline{u}(x)] = \Sigma_{i=1}^{N} w_i u_i(x) = \Sigma_{i=1}^{N} w_i [-m(x-x_i)^2+c_i]$

$= - m \Sigma_{i=1}^{N} w_i [(x-x_i)^2] + \Sigma_{i=1}^{N} w_i c_i$

$= - m \Sigma_{i=1}^{N} w_i [(x-x_i)^2] + C$ (a)

Differentiating with respect to x and setting to zero gives $x-\Sigma_{i=1}^{N}w_ix_i=0$. So the maximum social welfare is at the welfare ideal, $x^*=\Sigma_{i=1}^{N}w_ix_i$.

We now consider the first of the two terms on the right of the equation (a).

Note that $(x-x_i)^2 = (x-x^*+x^*-x_i)^2 = (x-x^*)^2+(x^*-x_i)^2 + 2(x-x^*) (x^*-x_i)$

Taking each of these three parts in turn we have:

$\Sigma_{i=1}^{N} [w_i(x-x^*)^2] = (x-x^*)^2$.

$\Sigma_{i=1}^{N} [w_i(x^*-x_i)^2] = V(\{x_i\})$, the 'population weighted variation'.

$\Sigma_{i=1}^{N} [w_i(x-x^*) (x^*-x_i)] = (x-x^*) \Sigma_{i=1}^{N} w_i(x^*-x_i) = 0$.

Theorem 6.3

Suppose that each individual has a quadratic utility function with individual sensitivity m_i. Then the welfare ideal x is the weighted sum of the individual ideals, the weights being a combination of the welfare weights and the sensitivities. Equation (7) still applies but with m, V and D redefined as given below.*

$x^* = \Sigma_{i=1}^{N} \omega_i x_i$	the weights are now ω_i
$\omega_i = m_i w_i / \Sigma_{i=1}^{N} m_i w_i$	where $\Sigma_{i=1}^{N} \omega_i=1$
$m = \Sigma_{i=1}^{N} m_i w_i$	the weighted mean sensitivity
$V(\{x_i\}) = \Sigma_{i=1}^{N} [\omega_i(x_i-x^*)^2]$	the weights are now ω_i
$D= (x-x^*)$	with $x^*=\Sigma_{i=1}^{N}\omega_i x_i$
$C = \Sigma_{i=1}^{N} w_i c_i$	as before

The proof follows the pattern of that for Theorem 2. Note that now the welfare ideal depends on the individual sensitivities so that the individual ideals of more sensitive individuals receive greater weight.

The multidimensional individuality utility function, with separable dimensions, analogous to equation (5) above, is:

$$u_i(x) = \Sigma_{j=1}^{M} W_j (- m_{ij}(x_j -x_{ij})^2 + c_{ij}) \qquad m_i\geq 0 \qquad (8)$$

Theorem 6.4

If the individual utility functions have the form given in equation (8), then the social welfare of a situation is the weighted sum of the one-dimensional social welfares *given in Theorem 2 above. The welfare ceiling is $C=\Sigma_{j=1}^{M}W_jC_j$. The multidimensional welfare ideal is the vector of the one-dimensional welfare ideals.*

$$U[\underline{u}(\underline{x})] = \Sigma_{j=1}^{M} W_j U_j$$
$$= \Sigma_{j=1}^{M} W_j [- m_j \{ V_j + D_j^2 \} + C_j]$$
$$= [\Sigma_{j=1}^{M} W_j [- m_j \{ V_j + D_j^2 \}] + \Sigma_{j=1}^{M} W_j C_j \qquad (9)$$

Finally it is worth noting that the specific results in this section are in accordance with the earlier remarks on the limitations to value. Here $v=(-m(V+D^2)+C)$; $v_{2max}=C$; and $v_{3max}=(-mV+ C)$; and the gaps due to sub-maximal ceilings, variation in ideals and sub-optimal design are: $g_{12}=(C - v_{1max})$; $g_{23}=-mV$; and $g_3=-mD^2$.

$$v = v_{1max}+ (C - v_{1max}) + ((-mV+ C)-C) + ((-m(V+D^2)+C) - (-mV+ C))$$

6.4 A practical application

The theory has discussed how the best design depends on individuals' values. In practice individuals' values may not be known and so a special investigation is needed to find out this information.

6.4.1 Ten dimensions of educational design

Quite recently I have started asking my colleagues at the Open University about the dimensions of educational design which are of particular concern to them. What has been interesting has been my colleagues' readiness to identify specific dimensions, and also the strength of their concern about these dimensions. They often seem to be saying: 'this is something I really want to sort out … this is an important choice that we need to get right'. Sometimes it is a choice between what they have been doing in the past and what they would like to do in the future. Sometimes it is a choice between what *they* want to do and what *their colleagues* want to do.

Firstly there is quite a lot of concern about student support. How much student support should there be? Should our contact with students be mainly or entirely by electronic means - or should we maintain our tradition of face-to-face contact? Should we be proactive in our contact with students – or should we wait for them to come to us? Given our limited resources, should these be devoted to provision of tutorials or to the provision of feedback on assignments?

Assignment policy itself is of course an extremely important aspect. What should the weighting be between tutor-marked and computer-marked assignments? Should we provide formative assignments as well as summative?

Student workload too is a critical area. Not least because we are anxious about our retention rates and are afraid that if we overload the courses the students will drop out.

Finally curriculum and teaching. Should we just give the students the content and let them get on with it – or should we provide some teaching? … and, if the latter, how much teaching should we give? Should we restrict the students to the topics we have selected – or should we allow them some freedom to choose topics to suit their own interests. In particular should we restrict the content to pure academic knowledge – or should we include content which relates to the student's current or future workplace?

These then are the dimensions which I have been looking at.

Display 6.1 Dimensions of educational design

Curriculum and teaching
- Curriculum balance
- Balance between content and teaching
- Freedom

Workload
- Weekly study time

Assignments
- Assignment policy
- The weighting of the Computer-Marked Assignment
- The percentage of assignments being formative
- The balance between tutor contact and assessment feedback

Tutorial support
- The amount of tutorial support
- The balance between face-to-face and other forms of tutorial support
- The number of proactive tutor contacts

6.4.2 One dimension

In order to illustrate my methodology I shall now look at the results for just one dimension, namely the provision of formative assignments. Formative assignments are those which the students are invited to do, but which are not compulsory and do not count towards the students' final grade. I designed a survey which contained the question shown in the slide. A sample of 200 students was sent the questionnaire and 65 students responded.

Display 6.2 The question about formative assignments

Imagine that you have the option to include in your course a proportion of formative assignments. Six options are listed below, ranging from 'no formative assignments' to '50% formative assignments'. Imagine your reactions to each of these options.
Please say on a scale of 0 - 10 how satisfied you think you would be to receive each of these options:
None of the assignments is formative
Around 10% of the assignments are formative
Around 20% of the assignments are formative
Around 30% of the assignments are formative
Around 40% of the assignments are formative
Around 50% of the assignments are formative

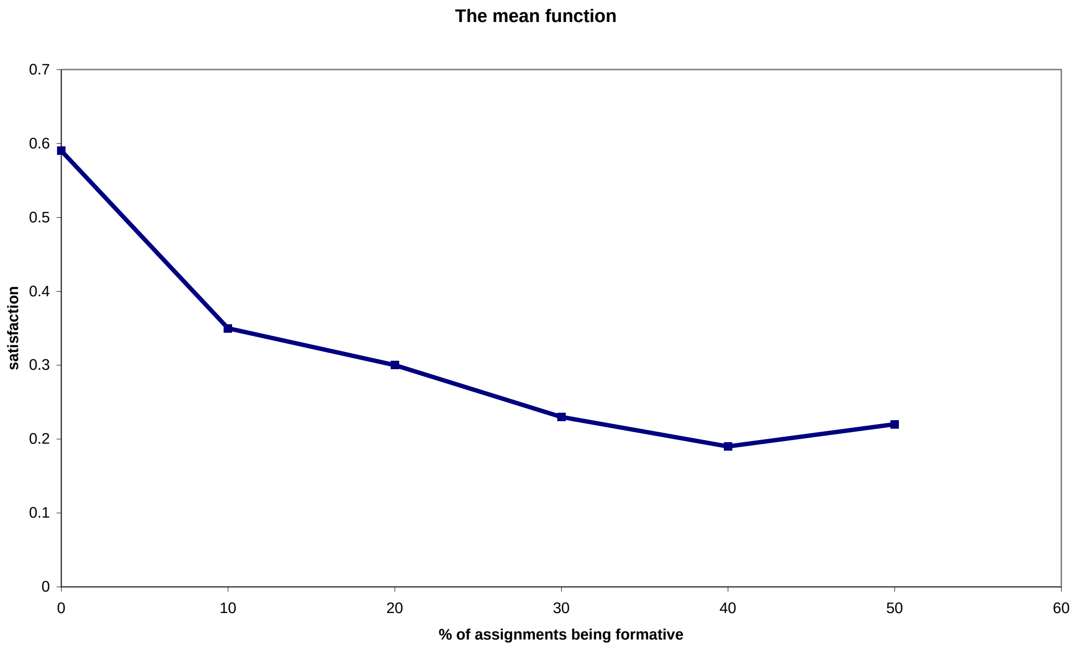
6.4.3 The results

My discussion of the results will be in eight parts – as shown in the slide. First I shall look at the response of the average student. Then I shall consider the differences which exist between students. Finally I shall look at the implications for educational design.

6.4.4 The average student: what does the mean function look like?

We start by looking at the mean function, in other words the means of the students' responses for each option. This is given in Figure 6.1 below. The horizontal scale gives the percentage of assignments being formative. The vertical scale gives the mean satisfaction and runs from zero to one. Zero is no satisfaction and one is extremely high satisfaction. If there are no formative assignments then satisfaction is 0.59 and this is the peak option. A quite sharp fall in satisfaction occurs if even just 10% of the assignments are formative. As the percentage of formative assignments rises the satisfaction continues to fall. The moderate level of peak satisfaction, 0.59, arises because students have different preferences. To see this we now look at the responses for three different students.

Figure 6.1 The mean function



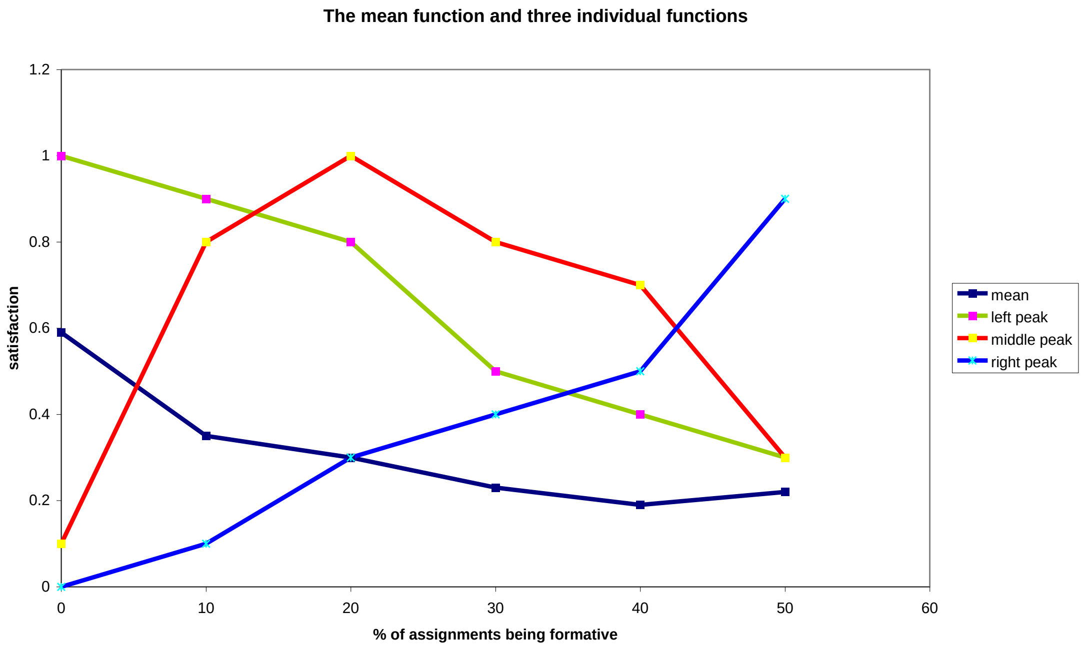
6.4.5 Differences between students

6.4.5.1 Three different students

One student (in green) prefers an option on the left; another student (in red) prefers a middle option; and a third student (in blue) prefers an option on the right. The consequence of this is that the mean function (in black) never reaches the peak satisfaction attained by any of the individual students.

Of course that's just three students and three peaks, a left peak, a middle peak and a right peak. Where are the peaks – the ideal points - for the other students?

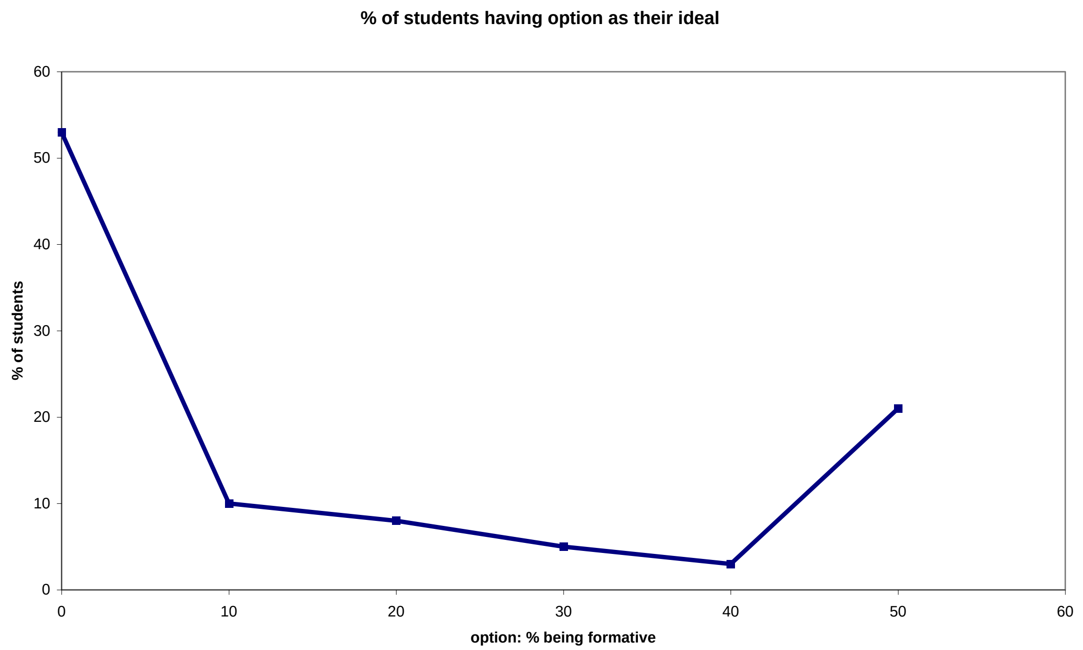
Figure 6.2 The individual functions for three students



6.4.5.2 Students have different ideals

Figure 6.3 presents the distribution of ideal points. For a majority of students, just over 50%, the ideal is to have no formative assignments. At the other extreme almost a quarter of students would like half of their assignments to be formative. Only a minority of students want any particular option in between the two extremes. These then are the ideal points of the individual students. However knowing only the ideal points tells us rather little about the shape of the individual student's responses. We shall refer to a student's responses as the utility function of that student.

Figure 6.3 The distribution of ideal points



6.4.5.3 What do the individual utility functions look like?

So, what do the individual utility functions look like? There are a number of possibilities. They might be strongly single-peaked or weakly single-peaked or they might have separate multiple peaks. A special case of strongly single-peaked is the point function.

Figure 6.4 Types of individual utility functions

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Strongly single-peaked      Weakly single-peaked
   x                                  xxx
  x x                               x     x
x     x                           x         x

Separate multiple peaks     Point function
   x    x                               x
  x x x x
x     x     x                     xxx  xxxxx
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It turns out that almost all (98%) of the students' utility functions are 'weakly single-peaked' - in other words once the function has started to fall it never rises. Furthermore 86% of the utility functions are strongly single-peaked (with the peak occurring at just one option) and 12% are flat-peaked (with the peak occurring at more than one option).

About a third of the students had point functions (i.e. a constant baseline and a single maximum point) …22% had point functions consisting of a constant baseline of zero and a single maximum point of one. In a further indication of the sharpness of some peaks, 46% of the students used none of the first four scale positions below their peak rating.

The results we have obtained so far have certain implications. In particular they impose limitations on the mean value of any option. As educational designers, in an ideal world we would want to provide an option with which everybody would be extremely satisfied.

6.4.6 Implications for educational design

6.4.6.1 Limitations on the mean value

However reality falls short of perfection! There are three distinct components to this. Let's start with perfection - a score of 1.0. The first problem is that even when an individual student is given their best option, still their satisfaction may fall short of 1.0. In fact the mean of individuals' maximum satisfaction is 0.83. The second problem is one which we have already noted. Different individuals have different ideal points. As a result even the best option has a mean satisfaction of only 0.59. The third problem is that a given option may not be the best option. If individuals are very unlucky they get the worst option and a mean satisfaction of only 0.19!

6.4.6.2 The social tension in the best option

So. Perfection is unattainable. The best we can do is to go for the best option. In other words we provide that option which elicits the highest mean satisfaction. Of course we cannot please all of the people all of the time. Some individuals will prefer an option other than the one we have chosen.

So there is social tension even in the best option. We can measure this social tension in a variety of ways. Let us consider the pair of options *zero formative assignment* (option A) and *10% of the assignments being formative* (option B).

One indicator of social tension is the proportion of people favouring either of the two options. Here 53% favour option A and 32% favour option B with 16% being neutral.

Another indicator of tension is the amount of satisfaction which people gain or lose depending on which option is selected. Those favouring option A stand to lose 0.63 if option B is chosen. Those favouring option B stand to lose 0.27 if option A is chosen.

An alternative indicator weights these results according to how many people are involved. *Overall mean satisfaction* is reduced by 0.33 when those favouring option A are provided with option B. *Overall mean satisfaction* is reduced by 0.09 when those favouring option B are forced to take option A. The gap between the satisfactions with these two options is 0.24.

6.4.6.3 How else might we define 'best'?

We have rather easily slipped into referring to the option with the highest mean satisfaction as the 'best' option. However there are a variety of other criteria which can be used for evaluating options and for choosing between them (Cowell 1995).

First consider voting procedures. With all options on offer at the same time, the zero option would take the most votes. In pairwise voting, the zero option would defeat all others, in other words it is the Condorcet winner. The median voter favours the zero option.

The zero option is a Pareto optimum. However every other option is also a Pareto optimum – illustrating the well-known weakness of the Pareto optimum as a selection criterion.

Turning to measures of inequality we find that the zero option is no longer always the most preferred. The 30% and 40% options are 'best' in terms of minimising range and standard deviation – and minimax. However the zero option is 'best' in terms of the coefficient of variation and maximin.

6.4.6.4 To increase satisfaction, increase choice

So: not only does the best option have only a modest mean satisfaction but also it is characterised by a fair amount of social tension and a high degree of inequality according to some measures. Is there not some way of doing better?

Well, yes there is – just as long as we are allowed to change our definition of the situation. Suppose that we are able to provide all the options. In that case individuals can choose their best option, and the mean satisfaction will then be increased to 0.83, the mean of the individuals' maximum satisfaction. However it may be that we cannot afford to provide all the options. How much can we boost satisfaction by providing just a few options? What the slide indicates is that offering three options provides almost as much mean satisfaction as offering all six options.

Display 6.3 Mean value depends on the number of options offered

One option	0.59
Two options	0.70
Three options	0.80
Four options	0.80
Five options	0.83
All six options	0.83