The Middle Opinion. USA 2020.

Chapter 13 (pre-election draft)[1]

Optimal social choice, preference functions:

Peter Emerson and Dublin City Council

Overview. Rosie Hackett was a member of the Irish Citizen Army in the Easter Rising of 1916 … How should Dublin commemorate its past? … Is it better to forget? … People disagree: they often have different values - so they often prefer different social options. How should a social choice be made? Different social choice methods sometimes choose the same option and sometimes choose different options. People tend to want the social choice method which chooses the option that they want. Losing sides often complain about the method used. This prompts the question: are some methods intrinsically better than others? A substantial literature addresses this question and this has led some people to advocate specific methods and to seek the adoption of these methods. Citing Dummett and others, Peter Emerson of the De Borda Institute[2] is a long-standing and energetic critic of two-option voting and of majority choices, advocating instead the Modified Borda Count (MBC) for decision-making, the Quota Borda System (QBS) in elections and the matrix vote in governance.

In 2013 Emerson was instrumental in Dublin City Council using the BC method to decide the name of a new bridge over the River Liffey. In this chapter, the results of the voting are analysed. Statistical measures of the distribution of rankings are used as criteria for judging the winner. Most methods choose the same winner, Rosie Hackett, but a few do not. Kay Mills has fewest lowest rankings and lowest polarisation. Rosie

Hackett is the most overtly political option, has a bimodal ranking distribution, has greater spread in rankings and has most polarisation. There is a sizeable negative correlation in the rankings of Hackett and Bermingham. A continuum from Hackett to Bermingham to Stoker exhibits single-peaked group means for 'supporter groups'. The corresponding single-peaked transversal has most voters on or near it. This Dublin Bridge continuum relates to the familiar Left-Right political continuum. Councillors' votes relate to their party allegiance. The Left vote for Hackett and the Right vote for Mills or Bermingham. The treatment of those voters who only give their top rankings is also discussed. Should the minimisation of polarisation be a social choice criterion? The social choice literature is noted.

Dublin City Council rank five names for the new bridge

On 2nd September 2013, fifty-one councillors of Dublin City Council voted, using a Borda voting method, to name a new bridge over the River Liffey. The name they chose was the Rosie Hackett Bridge. They used this method because Peter Emerson of the De Borda Insitute suggested they should. His first letter to them was sent after he saw a report in *The Irish Times* about the fact that they were going to name this new bridge, and that there were seventeen options 'on the table'. As far as is known, this is the first time that the Council, or indeed any elected chamber in Ireland, North or South, has used a Borda voting method in decision-making. The original seventeen options were reduced to just five options. In the following we have added a few words characterising each option:

RH   Rosie Hackett
         (socialist, trade unionist, Irish Citizen Army, woman)
KM Kay Mills (sportswoman)
WB Willie Bermingham (firefighter, campaigner)
FD   Frank Duff (founder of the Legion of Mary)
BS   Bram Stoker (author of *Dracula)*

Of these five, Rosie Hackett would seem to be the most controversial option, given her political stance as opposed to the somewhat apolitical aspects of the other options.

There were 51 councillors participating in the voting - one councillor was absent. A complete ranking of all five options was given by 39 councillors. An incomplete ranking was given by 12 councillors.

Table 1 presents the councillors' rankings of the five options. Each column gives the number of first preferences, second preferences, etc. given by the councillors for that option. Each column constitutes a frequency distribution of rankings for that option. All the distributions are asymmetric and all but one are unimodal – the exception being the Rosie Hackett option RH which is bimodal, with a mode at both extremes. This bimodality is consistent with the remark above that this person's name is politically more controversial.

In what follows, the rankings have been scored 4 for a first preference, 3 for a second, 2 for a third, 1 for a fourth and 0 for a fifth preference. So higher scores correspond to being more preferred. (Other scoring methods are discussed later in the chapter).

Table 1 The distribution of councillors' rankings of the five options

score	ranking	RH	KM	WB	FD	BS	ns
4	1st preferences	27	6	15	1	2	0
3	2nd preferences	7	25	11	1	6	1
2	3rd preferences	4	13	11	8	7	8
1	4th preferences	5	3	7	16	11	9
0	5th preferences	7	1	1	15	15	12
	not scored	1	3	6	10	10	30

Incomplete rankings; the 'not scored' entries

In Table 1 incomplete rankings are indicated by the 'not scored' row and the 'ns' column. Altogether there were 30 not scored entries. The final column in Table 1 provides some information about the incomplete rankings which were given by 12 councillors. None of the 'incomplete' councillors gave a last (5th) preference (ns=12). A minority of the 'incomplete' councillors gave a second last (4th) and third last (3rd) preference (ns=9 and ns=8). All gave a first preference (ns=0) and all but one gave a second preference (ns=1). Broadly speaking the majority of the 'incomplete' councillors stated their first two preferences and left everything else blank, giving no information about the ordering of the remaining three options.

This would suggest that the least preferred options account for most of the not scored entries. This is indeed the case. Consider first preferences. 'FD' and 'BS' were the two least preferred options; and they accounted for 20 out of the 30 not scored entries. 'RH' is the most preferred option; and it accounts for just one of the not scored entries.

It also would suggest that the options with distributions that are more spread out account for more of the not scored entries. Thus the 'WB' distribution is more spread out than the 'KM' distribution and accounts for more of the not scored entries.

The treatment of incomplete rankings

We now consider the treatment of incomplete rankings. The approach adopted for the analysis in the first part of the chapter is to estimate the missing entries as follows:

> If a voter gives their first four preferences but not their fifth preference then the remaining option is scored '0'.
> If a voter gives their first three preferences but not their fourth or fifth preferences then the remaining options are scored '0.5'.
> If a voter gives their first two preferences but not their third to fifth preferences then the remaining option are scored '1'.
> If a voter gives their first preference but not their other preferences then the remaining option are scored '1.5'.

The Modified Borda Count uses a difference approach motivated by its attention to the social choice process. It downgrades the scores for councillors giving incomplete rankings – see discussion later in the chapter.

The analysis of the results

Our analysis starts with measures of central tendency: the mean, median and mode. If the distribution of scores is symmetric the mean equals the median. If the distribution of scores is symmetric and unimodal the mean and median both equal the mode. Even when these conditions are not met it can be the case that there is approximate equality. Note that the Borda Count is the *sum of ranks* and – for the purposes of comparing options - is equivalent to the *mean rank* (if there are no unscored entries).

Table 4 presents the mean, median and mode ranking for each option. Each of options RH, KM and WB have a high mean, median and mode. Option RH, Rosie Hackett, is the highest – by a small amount. Because it

has the highest mean, option RH is the Borda Count winner. In contrast each of options FD and BS has a low mean, median and mode.

The final row gives what I have called the anti-mode, namely the score with lowest frequency. 'RH' has an anti-mode of 2 and a mode of 1 and 4.

Table 4 The mean, median and mode score for each option
An * indicates the winner according to the given criterion

	RH	KM	WB	FD	BS
mean	2.80*	2.58	2.52	0.92	1.18
median	4*	3	3	1	1
mode	4*, 1	3	4*	1	0
anti-m.	2	0	0	4	4

We now consider two measures of spread: the range and the standard deviation. Here the full range of rankings appears for all five options. The standard deviation is greatest for Rosie Hackett. This too is consistent with the point above that this person's name is politically more controversial. See Table 5.

Table 5 The standard deviation of the scores for each option

	RH	KM	WB	FD	BS
standard deviation	1.5	0.9	1.2	0.9	1.1

We now consider the cumulative frequency distribution. Table 6 below gives for each option, the number of voters who have: that option as their first preference; that option as their first or second preferences; that option as their first, second or third preferences; that option as their first, second, third or fourth preferences; … that option as their last preference; and that option as their last or second last preferences.

These different criteria relate to two different notions about what constitutes a good choice. One notion is that a good choice should be one

that a lot of people rank highly. Another notion is that a good choice should be one that few people rank lowly.

Rosie Hackett has the most first preferences; and also the most first two preferences. However Kay Mills has the most first three preferences; and the most first four preferences (jointly with WB) ... and the least last preferences (jointly with WB); and the least last two preferences.

Table 6 The cumulative frequency distribution for the scores
An * indicates the winner according to the given criterion

.	RH	KM	WB	FD	BS
first	27*	6	15	1	2
first two	34*	31	26	2	8
first three (v. last two)	38	44*	37	10	15
first four (v. last)	44	50*	50*	35	34
last two	13	7*	14	41	36
last	7	1*	1*	16	17

We now consider pairs of options. Given two options J and K we can count the number of people who prefer J to K and the number of people who prefer K to J. This enables us to say that J is pairwise preferred to K or vice versa. This is the basis for Condorcet voting. Emerson reports that RH wins over KM which wins over WB which wins over BS which wins over FD. So option RH, Rosie Hackett, is the Condorcet winner.

Another measure is the mean difference between the rankings for J and for K. However this is the same as the difference between the means of J and K – which are reported in Table 2 above – and leads to the same ordering of options as the Borda Count.

Polarisation can be said to occur if an option is ranked highly by many voters but also ranked lowly by many voters. One measure of this might be the proportion of first preferences times the proportion of last preferences.[3] In this case Rosie Hackett has a much higher polarisation score than any other option. Again this is consistent with earlier remarks about this option being controversial. See Table 7. For example RH=0.076=(27/51)(7/51).

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[3] This measure is used in Burt, 207, op. cit. 222, 221-226, where the actual outcome of a Conservative government is least negative but most polarising.

Table 7 The polarisation scores for each option

	RH	KM	WB	FD	BS
polarisation	0.073	0.002	0.006	0.007	0.012

Do different social choice methods choose different options? Yes, they do. What we have found is that the Borda count (highest mean), highest median, first preference (majority), first two preferences and Condorcet methods all choose Rosie Hackett as the winner; but the first three preferences (last two preferences) and first four preferences (last preference) choose Kay Mills as the winner. The highest mode chooses Rosie Hackett and William Bermingham jointly. Finally Kay Mills has the lowest polarisation (and Rosie Hackett has the highest).

The correlation matrix

Table 8 presents the correlations between rankings, based on the 39 councillors who gave a complete set of rankings. The largest correlation in magnitude is the correlation of -0.52 between Hackett and Bermingham. High scores for Hackett are associated with low scores for Bermingham and vice versa. The options are ordered according to the size of the correlation with Hackett. Note that in general the correlations decrease, the farther away the entry is from the main diagonal.

Table 8 The correlations between option scores

	RH	KM	FD	BS	WB
Rosie Hackett	1	-0.17	-0.30	-0.39	-0.56
Kay Mills		1	-0.14	-0.31	-0.15
Frank Duff			1	-0.14	-0.12
Bram Stoker				1	-0.12
Willie Bermingham					1

A somewhat unusual feature of the correlation matrix is that all the correlations are negative. This happens because the data concerns

rankings: the sum of each person's rankings is a constant; and this gives a negative relationship between the variables.

RH + KM + WB + FD + BS = 15
RH = 15 - KM - WB - FD – BS

Single-peaked preferences

Taking the continuum introduced in the previous section, are preferences single peaked? First let us consider group preferences.

We divide the voters into five groups depending on their first preferences: the Hackett supporters; the Mills supporters; the Bermingham supporters; the Duff supporters; and the Stoker supporters. Figure 1 below presents the means of the rankings given by the Hackett group – and the means of the rankings given by each of the other four groups. Each group has single-peaked preferences (except where Bram Stoker is involved ... note too that the Stoker curve itself is based on just two voters). So *group* preferences are single-peaked – with just a few exceptions. Note that the options have been ordered on the continuum in such a way as to exhibit single-peakedness.

Figure 1 Group mean scores: group preferences are single-peaked

mean of a group's ranking
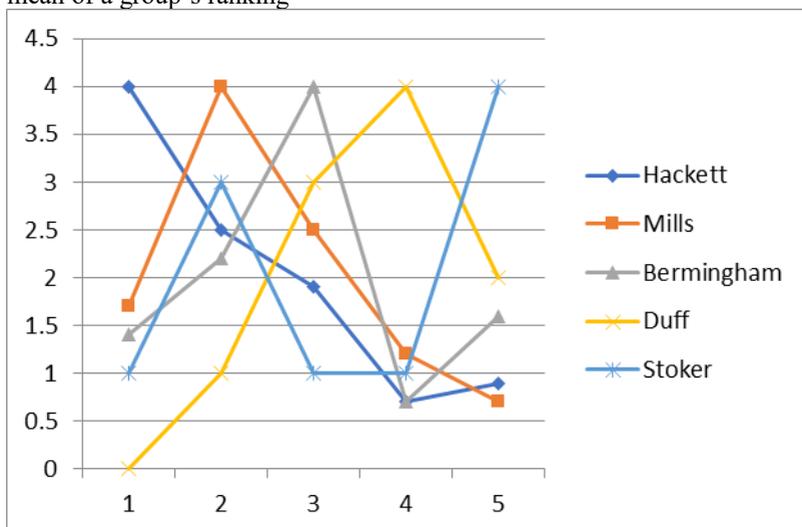
Are *individual* preferences single-peaked? More precisely, can the options be ordered on a continuum in such a way that individual preferences are single peaked.

Consider first, just the three options: Hackett, Mills and Bermingham. There are six possible preference orderings: HMB, MHB, MBH, BMH, BHM and HBM. In that order they can put in to a ring to form a preference hexagon where each adjacent pair is such that one can be produced from the other by a single adjacent transposition. We refer to HMB-MHB-MBH-BMH as a transversal of the preference hexagon.

There are three possible continua: H in the middle (B to H to M or the reverse), M in the middle (B to M to H or the reverse) or B in the middle (H to B to M or the reverse). Of 37 individuals, 25 are consistent with H in the middle, 30 are consistent with M in the middle, and 19 are consistent with B in the middle. So the data fits M in the middle best. This is consistent with the continuum in Figure 1 above. Using this continuum the 30 individuals gave single-peaked rankings HMB 13, MHB 5, MBH 3 and BMH 9. Contrary to single-peakedness on this continuum were HBM 5 and BHM 2. We refer to HMB-MHB-MBH-BMH as a single-peaked transversal of the preference hexagon. Most people (thirty) were on the transversal and just some (seven) were not.

Now consider all five options. There are 5!=120 possible preference orderings. They can put in a preference polyhedron in four dimensions where each adjacent pair of rankings is such that one can be produced from the other by a single adjacent transposition. The polyhedron has transversals some of which are consistent with single-peakedness.

Placed on the HMBDS continuum, 11 people have single-peaked rankings; 19 people have rankings just one adjacent transposition away from single-peaked; 5 people just two adjacent transpositions away; and one person each for three, four, five, and six adjacent transpositions away. So most people (30) were on or next to the single-peaked transversal and just some (9) were not. See Table 9.

Table 9 The number of rankings at distances from the single-peaked transversal HMBDS.

Number of adjacent transpositions away from single-peaked transversal HMBDS						
0	1	2	3	4	5	6
No. of rankings 11	19	5	1	1	1	1

The Dublin Bridge continuum and the Left-Right continuum

The previous discussion has led to the notion of a Dublin Bridge continuum. We now ask how this relates to the familiar Left-Right political continuum. This is a fairly natural question to ask given that the winner Rosie Hackett has a quite explicit position on the Left and that this seemed to relate to certain features of her rankings. We address this question by looking at how councillors' votes related to their party allegiance.

There are fifty-two councillors on Dublin City Council, representing seven different political parties plus a number of independents: Fianna Fáil (6), Fine Gael (12), independents (8), Labour (18), PBPA (1), United Left (1), éirígí (1) and Sinn Féin (5). This ordering of parties is similar to the ordering of parties identified in an earlier chapter as the principal component in the historical variation in party votes. The ordering would appear to relate to a Left-Right political continuum (with the left somewhat associated with contemporary Irish nationalism).

Table 10 presents the relationship between political party and the Dublin Bridge vote. Voting was almost entirely along party lines: 92% of the Left voted for Hackett but 0% of Fine Gael and Fianna Fáil did; and just 4% of the Left voted for Bermingham but 56% of Fine Gael and Fianna Fáil did. Independents split 50% and 50% between Hackett and Bermingham. A majority of Fine Gael supported Willie Bermingham with just one person supporting Kay Mills. A majority of Fianna Fáil supported Kay Mills with just two persons supporting Willie Bermingham.

Thus the Dublin Bridge continuum correlates highly with the familiar Left-Right political continuum.

Table 10 The relationship between political party and first preferences

party .	councillors	first preferences				
		RH	KM	WB	FD	BS
Fianna Fáil	(6)	0	**4**	**2**	0	0
Fine Gael	(12)	0	1	**8**	1	2
Independents	(8)	**4**	0	**4**	0	0
Labour	(18*)	**15**	1	1	0	0
PBPA	(1)	1	0	0	0	0
United Left	(1)	1	0	0	0	0
éirígí	(1)	1	0	0	0	0
Sinn Féin	(5)	**5**	0	0	0	0

* One councillor was absent.

Polar-Min social choice?

This section is very much an afterthought, a somewhat troubled afterthought …

Most methods – absolute majority, top few places, Condorcet, Borda Count, some of the methods that experts have identified as the best - choose the same winner, Rosie Hackett …

… but a few methods do not. Kay Mills has fewest lowest rankings and lowest polarisation. Rosie Hackett is the most overtly political option, has a bimodal ranking distribution, has greater spread in rankings and has most polarisation. There is a sizeable negative correlation between the rankings of Hackett and Bermingham. Councillors' votes relate to their party allegiance. The Left vote for Hackett and the Right vote for Mills or Bermingham.

Suppose that what is important is the avoidance of extremes and the avoidance of polarisation. Perhaps that option should be chosen which minimises polarisation. Perhaps Kate Mills should have been the option chosen. Perhaps 'Polar-Min' – minimising polarisation - is the best social choice method.

This measure is used in Burt, 207, op. cit. where the actual outcome of a Conservative government has the highest satisfaction but is most polarising.

This suggestion certainly fits current concerns among liberals about the rise of polarisation and extremism. It also relates to a recent article by Cahan and Slinko who describe a method where "none of the most extreme candidates receive the most electoral support" – see the abstract of their paper at the end of the chapter.

A tolerant political culture

A central strand in Peter Emerson's argument is the divisiveness caused by voting when it uses certain 'bad' social choice methods. I suggest a possible recasting of this as an intolerance of the results produced by certain 'bad' social choice methods.

How tolerant would Rosie Hackett supporters be of the result produced by a Polar-Min method? "What!? We've won according to almost every known social choice method and you are choosing some other option!" How tolerant are Rosie Hackett opponents of the result produced by the Borda Count method? I don't know. Maybe: "What a surprise! You are in the majority and you are imposing your majority preferred option on us despite that fact that it's the one we least prefer."

It would appear to be difficult to find a method that is not open to objection. So there is value in having a tolerant political culture.

The number and popularity of options – a geometric series?

As noted, the original seventeen options were reduced to just five options. This was done by the six-member 'naming committee' using their first BC. (Thus the main ballot by all 51 councillors was their second use of the BC methodology.) At an even earlier stage of the process there had been many more proposed names.

This prompts us to consider how many options there are in the full set of options envisaged by the population - and what the distribution of popularities is of the options. In many situations what happens is that there are a few popular options and many unpopular options.

Why does this occur? It may be that options can be located in an option space and that the population has a unimodal distribution of individual ideal points in that option space. The few options near the mode are popular and the many options far from the mode are unpopular.

Elsewhere I have used a geometric series to represent an approximation to the distribution of popularity of parties. Table 11 shows

that the number of first preferences in order approximate the geometric series $p_n=0.5(0.5)^n$

Table 11 The distribution of the number of first preferences, $p_n=0.5(0.5)^n$

	most popular				least popular
no. of first preferences	27	15	6	2	1
data (proportion)	52.9	29.4	11.8	3.9	2.0
model (proportion)	50.0	25.0	12.5	6.3	3.1

Links … the social choice literature

The best link to Peter Emerson's work is his 2012 book *Defining Democracy, Voting Procedures in Decision-Making, Elections and Governance,* Springer, 2012.[4] Also noted in this chapter have been his articles in *Representation* and *Social Choice and Welfare*. Many of his writings can be accessed through the website of the de Borda Institute.

A key idea for Emerson is that voters should be offered more than two options – and that each voter should give their full set of preferences. In 2014 he commissioned a survey to establish the views of Scottish voters on six options for Scotland in relation to Scottish independence (or not). The 2014 Yearbook contains a further analysis of the results.[5] In similar vein, in 2015 YouGov ran a survey asking Labour party members for their full set of preferences for the party leadership. The 2015 Yearbook contains a further analysis of the results.[6]

Another idea which I have been interested in is the fact that economists sometimes use preferences and sometimes use utilities. In politics, data on preferences is more common than data on utilities. Yet the latter provides us with more precise information. In the run-up to the UK general election of 2015, it was thought that no party would have an overall majority.

[4] Emerson, Peter. *Defining Democracy, Voting Procedures in Decision-Making, Elections and Governance,* Springer, 2012.
[5] Burt, Gordon. *Values, World Society and Modelling Yearbook, 2014*. Newcastle: Cambridge Scholars, 2016. 193-195.
[6] Burt, Gordon. *Values, World Society and Modelling Yearbook, 2015*. Newcastle: Cambridge Scholars, 2017. 235-243.

YouGov ran a survey asking for a rating of the value of different governing party(-ies) options. The 2015 Yearbook contains a further analysis of the results.[7]

A foundational approach to these ideas is taken in my book, *Conflict, Complexity and Mathematical Social Science*.[8] The book covers models of individual choice in psychology (Chapter 9), preferences and utilities in microeconomics (Chapter 12), social choice and social welfare (Chapter 4), single-peaked preferences (Chapter 5), social value functions (Chapter 6), rationality, rules, religion or randomness (Chapter 10) and social ideals and power (Chapter 15).

A major reference work is the *Handbook of Social Choice and Social Welfare*.[9] Amartya Sen has recently revised his book *Collective Choice and Social Welfare*:[10]

> *Nobel Prize winner Amartya Sen's first great book, now reissued in a fully revised and expanded second edition*
> 'Can the values which individual members of society attach to different alternatives be aggregated into values for society as a whole, in a way that is both fair and theoretically sound? Is the majority principle a workable rule for making decisions? How should income inequality be measured? When and how can we compare the distribution of welfare in different societies?'
> These questions, from the citation by the Swedish Academy of Sciences when Amartya Sen was awarded the Nobel Memorial Prize in Economics, refer to his work in Collective Choice and Social Welfare, the most important of all his early books. Originally published in 1970, this classic work in welfare economics has been recognized for its ground-breaking role in integrating economics and ethics, and for its influence in opening up new areas of research in social choice, including aggregative assessment. It has also had a large influence on international organizations, including the United Nations, particularly in its work on human development. In its original version, the book showed that the 'impossibility theorems' in social choice theory-led by the pioneering work of Kenneth Arrow need not be

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[7] Burt, 2017, op. cit., 221-226.

[8] Burt, Gordon. *Conflict, Complexity and Mathematical Social Science*. Bingley: Emerald. 2010.

[9] Arrow, K. J., Amartya K. Sen and K. Suzumura. *Handbook of Social Choice and Social Welfare*. Amsterdam, Elsevier: 2002.

[10] Sen, Amartya. *Collective Choice and Social Welfare*. Revised edition. Penguin: 2017. Accessed, 4 February 2018.
https://www.penguin.co.uk/books/290088/collective-choice-and-social-welfare/#q5f7MZolWESoyb8l.99.

seen as destructive of the possibility of reasoned and democratic social choice.

Sen's ideas about social choice, welfare economics, inequality, poverty and human rights have continued to evolve since the book's first appearance. This expanded edition, which begins by reproducing the 1970 edition in its entirety, goes on to present eleven new chapters of new arguments and results. As in the original version, the new chapters alternate between non-mathematical chapters completely accessible to all, and those which present mathematical arguments and proofs. The reader who prefers to shun mathematics can follow all the non-mathematical chapters on their own, to receive a full, informal understanding. There is also a substantial new introduction which gives a superb overview of the whole subject of social choice.

Finally an article[11] in the latest issue of *Social Choice and Welfare*[12] addresses the central concerns of the present chapter. As in this chapter, the authors pay attention to multiple candidates, spatial election models, most preferred and least preferred candidates, and concern to avoid extreme winners.

> We characterise multi-candidate pure-strategy equilibria in the Hotelling–Downs spatial election model for the class of best-worst voting rules, in which each voter is endowed with both a positive and a negative vote, i.e., each voter votes in favour of their most preferred candidate and against their least preferred. The importance of positive and negative votes in calculating a candidate's net score may be different, so that a negative vote and a positive vote need not cancel out exactly. These rules combine the first-place seeking incentives of plurality with the incentives to avoid being ranked last of antiplurality. We show that, in our simple model, arbitrary best-worst rules admit equilibria, which (except for three candidates) are nonconvergent if and only if the importance of a positive vote exceeds that of a negative vote. The set of equilibria in the latter case is very similar to that of plurality, except the platforms are less extreme due to the moderating effect of negative votes. Moreover: (i) any degree of dispersion between plurality, at one extreme, and full convergence, at the other, can be attained for the correct choice of the weights; and, (ii) when they exist (and there are at least five candidates), there always exist nonconvergent equilibria in which none of the most extreme candidates receive the most electoral support.

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[11] Cahan, Dodge and Arkadi Slinko. "Electoral competition under best-worst voting rules." *Social Choice and Welfare*. 2018. Accessed, 4 February 2018. https://doi.org/10.1007/s00355-018-1115-7.

[12] *Social Choice and Welfare*. Accessed, 4 February 2018. https://link.springer.com/journal/355.