

Brexit: the VFSD model

Value Function Spatial Distribution

Key point

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Key point

The hypothesis is the following. The set of Brexit options can be represented by the space, $[0,1]$. Voters' ideal points for Brexit have a uniform distribution over Brexit space, $[0,1]$. The best option in terms of highest mean value and lowest polarisation is at the mean/median point $x=0.5$, exactly on the borderline between Leave and Remain.

The data from various studies suggests that the uniform distribution provides a fair approximation with the mean or median possibly within the range $[0.4,0.5]$ – in other words an amount 0.1 either side of the Leave-Remain borderline. Those Brexit options which are between May's Deal and the Leave-Remain borderline (in other words the agenda of the May-Corbyn talks) are somewhere in the interval $[0.3,0.5]$.

1 Surface percentages and underlying reality

Surface behaviour may be an expression of an underlying reality. A surface percentage may be an expression of an underlying distribution of individuals in space. The distribution can be characterised by a mean and a standard deviation (or some other measures of average and of spread). There is a variety of social choice criteria, some percentage-based, some space-based. In particular the mean point is an appealing rendering of the phrase ‘what people want’. In some cases the mean point may be the optimal social choice, maximising mean value and minimising value spread (polarisation).

2 Two opposing sides:

2.1 Remain or Leave?

The point of the 2016 referendum was to allow the British people to have their say: did they want to Remain in the European Union or did they want to Leave? In the event they voted to Leave, 52% to 48%. That was ‘what the people said’ ... that was ‘the people’s choice’ ... that was ‘what the people wanted’ ... that was ‘the will of the people’¹.

Since the referendum, opinion polls have continued to ask the question, Remain or Leave? In Spring 2019 and for some time now, the opinion polls give a somewhat different answer. The Brexit poll of polls gives Remain 54% and Leave 46%. ‘What the people want’ has changed.

<https://whatukthinks.org/eu/opinion-polls/euref2-poll-of-polls/>

‘The people want X’. Etc. I prefer not to use these phrases. I think the phrase ‘the people want X’ can mislead. It can mislead in two ways. Firstly it might be thought to refer to all the people whereas it may be that just a certain percentage of the people want X. Secondly X may be unspecific. ‘X’ may refer not to just one specific thing but rather to any one of a number of things – in other words X is a set of things. Different people may want different things from the set X.

Thus, after the 2016 referendum, the question arose: what did Leave mean? Prime Minister Theresa May explained: ‘Leave’ means ‘Leave’. As the debate unfolded several proposed variants of ‘Leave’ appeared. Indeed a continuum was implied with references to a hard or soft Brexit and varying degrees of hardness or softness. Prime Minister Theresa May thought that her Deal meant ‘Leave’ while others thought it did not – it was ‘Brino’: Brexit in name only.

All this suggests that, although the voters in the referendum were forced to select between just two options, there was in fact an underlying reality: the Brexit continuum of options. We think of people as being distributed along a one-dimensional continuum, with varying degrees of commitment to Leave or Remain and

¹ *The Will of the People: a Modern Myth*. Albert Weale. Cambridge, Polity Press: 2018. <https://www.amazon.co.uk/Will-People-Modern-Myth/dp/1509533265>

a boundary point between Leave and Remain. These basic ideas are at the core of the VFSD model: Value Function Spatial Distribution.

2.2 The VFSD model for two options

Consider the case of a binary choice between two options such as a referendum or an opinion poll about Brexit. The surface situation is fairly simple. However it may be that the underlying reality is more complex. One possibility is that the VFSD model may apply.

The surface situation is as follows. There are two options: Remain or Leave. Each individual is asked whether they want to Leave or Remain in the EU. The surface behaviour is that each individual answers 'Leave' or 'Remain'. This gives the surface percentages, P% Remain and Q=(100-P)% Leave.

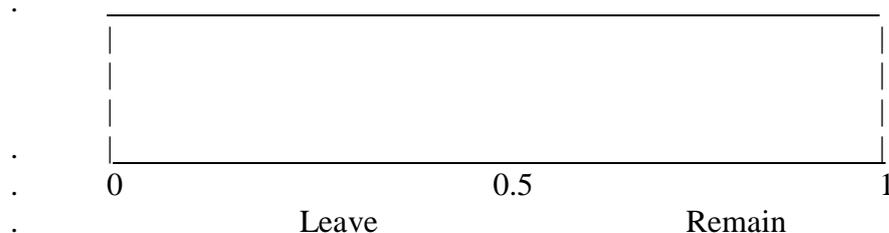
In the 2016 referendum, P=48 and Q=52. In the Spring of 2019, opinion polls suggested P=54 and Q=48. To begin with however, let us assume for simplicity that P=Q=50%. We shall return to the real figures shortly.

Taking this as the surface situation we now reconceptualise it in terms of the VFSD model. Suppose that the underlying reality is that there are not two options but a continuum of options, different degrees of hard or soft Brexit. Let us represent this continuum by a number x that can take any value between 0 and 1 - between the most extreme form of Leave and the most extreme form of Remain. The borderline between Leave and Remain is at x=0.5. Below 0.5 is Leave; and above 0.5 is Remain.



Each individual wants a certain amount x of Leave/Remain. This is their ideal amount, the amount with the most value, their first preference. If x is below 0.5, the person chooses Leave; and if x is above 0.5, the person chooses Remain.

People are distributed along the Brexit continuum – that is, their values of x are distributed along the continuum. In this example, for simplicity, we suppose that they are evenly distributed along the continuum – what is known as the uniform distribution.



Because the distribution is even – because it is a uniform distribution – there are as many people below 0.5 as there are above 0.5 – it's half and half. In other words the results of a poll or a vote would be 50% Leave and 50% Remain. This illustrates a fundamental point:

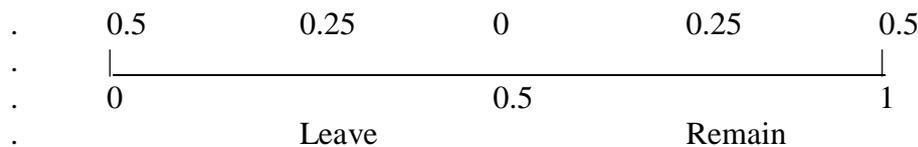
The surface percentages are an expression of the underlying distribution.

Statisticians usually characterise a distribution in terms of a mean and a standard deviation (or some other measures of average and spread). In this very simple case we can say what these are by inspection. The mean is in the middle at 0.5. (Note that in this case the median and the mean are the same.)

How spread out are people from the mean? How distant are people from the mean? Some are exactly in the middle and some are at the extreme, a distance of 0.5 away. On average people are halfway away: 0.25 distant from the middle. We shall refer to this measure as ‘spread*’ (the more technical term is ‘mean deviation’).

In summary, mean=median 0.5; spread*=0.25.

distance from middle:



Notice that the question we asked was: how spread out are people *from the mean*? A different question is: how spread out are people *from the point $x=0$* ? How distant are people from the point 0? Some are at the far extreme, a distance of 1.0 away and some are exactly in the middle, a distance of 0.5 away. On average people are 0.5 distant from the point $x=0$. We shall refer to this measure as ‘the spread from $x=0$ ’ (the more technical term is ‘mean deviation about $x=1$ ’). We can write $s(0)=0.5$. By symmetry, it is also the case that the spread from $x=1$ is $s(1)=0.5$.

The general question is: how distant are people from the point x ? It can be shown that what might be called the point x -spread is $s(x)=(x^2+(1-x)^2)/2$.

The x -spread has a maximum of $s(x)=0.5$ at the extremes $x=0$ and $x=1$; and a minimum of $s(x)=0.25$ at the middle $x=0.5$. In other words:

For the uniform distribution, the mean is the point at which the x -spread is at its lowest. (The standard result is formulated in terms of the variance: the mean is the point at which the sums of squares of the distance from the point is a minimum.)

In terms of social value, we might interpret the mean point 0.5 as ‘what people want’ and the spread* as minimising the polarisation. Thus the single point $x=0.5$, is the best in terms of two different criteria.

[In some cases, it may be the optimal social choice, maximising mean value and minimising value spread (polarisation).]

This provides a rationale for considering the middle option as being particularly desirable.

Options as points ... or intervals with boundaries ... or mid-points of intervals

We started off with the two surface options of Remain and Leave. How should we think of these options spatially – in relation to the Brexit continuum? We can think of these two options in terms of the Brexit continuum in various ways – either as points or as intervals with boundary points or as the mid-points of intervals.

Consider the Leave-Remain Brexit dimension. Consider just the two options: Leave, Remain. Suppose we assign scores to the options:

Two points:
0 and 1

... however an option can have different expressions and so it may be more appropriate to represent an option as occupying an interval along a dimension, with boundary points between the intervals.

Two intervals on the Leave-Remain dimension - with one boundary point $x=0.5$:
[0,0.5] [0.5,1.0]

Sometimes it is useful to think of options being at the mid-points of their intervals. We shall do this in our calculations later.

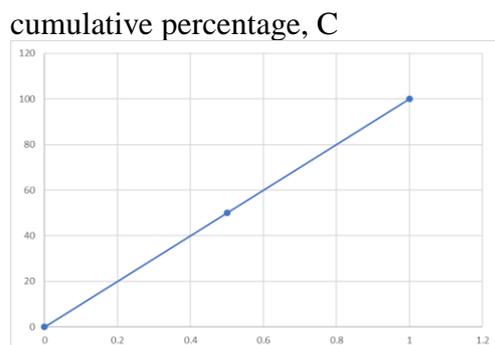
Two midpoints of intervals on the Brexit Leave-Remain dimension:
0.25, 0.75
[0,0.5] [0.5,1.0]

The cumulative distribution

We have already said that half the people vote Leave, in other words half (50%) are below the midpoint 0.50. Indeed because the distribution is uniform it is in general the case that: $x\%$ of the people are located at x or below x . This relationship between the 'cumulative' percentage C and the position x is referred to as the cumulative distribution. It increases immediately from $C=0\%$ at $x=0$ and reaches $C=100\%$ at $x=1.0$. See Figure 1.

The median is when $C=0.5$, namely at $x=0.5$. The first quartile is when $C=0.25$, namely $x=0.25$. The third quartile is when $C=0.75$, namely $x=0.75$. So the interquartile range, another (different) measure of spread, is 0.50. Half the interquartile range (HIR) is 0.25. We refer to this as the HIR-spread.

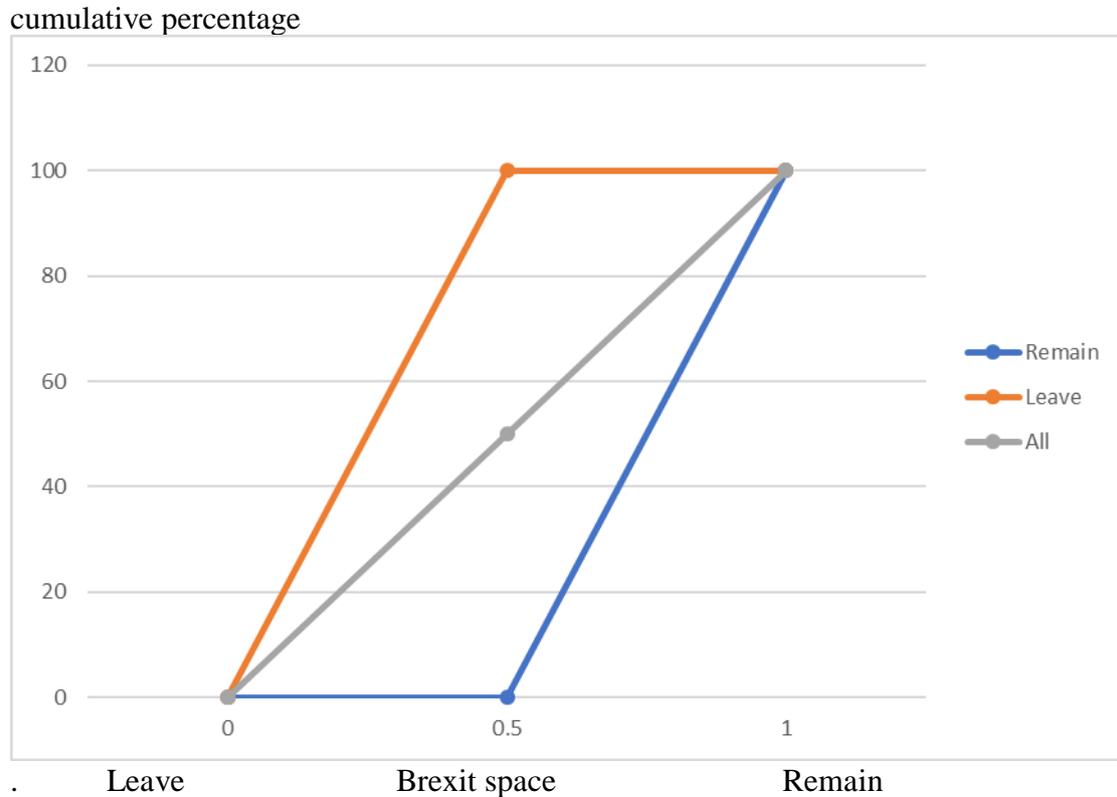
Figure 1 The cumulative distribution for all the people; $C=x$



.Leave Remain x

We can also think about the separate cumulative distributions for Leavers and Remainers. For Leavers C increases immediately from 0% at x=0 and reaches 100% at x=0.5. For Remainers C is flat until 0.5 and then increases from 0% at x=0.5 and reaches 100% at x=1.

Figure 2 The cumulative distributions: for Leavers, for Remainers and for all



In summary the uniform distribution has the following properties:

mean 0.50
 median 0.50

HIR-spread 0.25
 *spread 0.25
 x-spread $s(x)=(x^2+(1-x)^2)/2$ is a minimum at the mean $x=0.5$

consensus/ polarisation
 polarisation A *spread spread of the distribution from the mean

... as spread over the continuum:

consensus 0.04
 polarisation B 0.96

The 2016 referendum

We now return to the real figures for the 2016 referendum, $P=48$ and $Q=52$. On the surface level, the mode is Leave with 52%.

In terms of the VFSD model we have:

mode	Leave, [0,0.5] ... midpoint 0.25
mean*	0.48, just a little bit on the Leave side of the boundary point.
mean*	0.49, just a little bit on the Leave side of the boundary point.
median	0.48

HIR-spread 0.25

consensus/
polarisation ... as spread over the continuum:

consensus	0.04
polarisation	0.96

Notes:

.(i) mean: 0.48 if scores of 0 and 1 are used; 0.49 if scores of 0.25 and 0.75 are used.

The first calculation of the mean point makes an assumption, namely that all the people are at 0 or 1. This contradicts our statement that people are scattered along the continuum. Suppose the mean point of Leave votes is L and the mean point of remain voters is R . Then the overall mean is $qL+pR=L+p(R-L)$. For example, taking $L=1/4=0.25$ and $R=3/4=0.75$ gives a mean of $0.25+p/2$. This gives the second calculation.

.(ii) median: $y=0.5$ and $y=1.04x$; so $x=0.48$; x is Brexit score and $y=f(x)$ is cumulative distribution function.

.(iii) Consensus/polarisation. There are two distinct concepts in play here.

One concept is whether there is consensus around one or other of the options, whichever is most supported. A measure of consensus, ranging between 0 and 1, is $|p-q|$. Here, with $p=0.48$ and $p=0.54$, consensus is very low at $c=0.04$ and $c=0.08$; and polarisation very high at $(1-c)$.

A quite different notion of consensus/polarisation is the variation of the distribution of individuals over the continuum. One measure of this might be the difference in the means of the two groups, $R-L$. This is quite unrelated to the proportions p and q .

The Spring 2019 opinion polls

We now return to the real figures for the Spring 2019 opinion polls, $P=54$ and $Q=46$. On the surface level the mode is Remain with 54%.

In terms of the SVF model we have:

mode:	Remain, [0.5, 1.0] ... midpoint 0.75
mean*:	0.54, just a little bit on the Remain side of the boundary point.
mean*:	0.52, just a little bit on the Remain side of the boundary point.
median:	0.54

consensus/ polarisation	...	as spread over the continuum
consensus	0.08	... around one option
polarisation	0.92	... around one option

Notes:

.(i) mean: 0.54 if scores of 0 and 1 are used; 0.52 if scores of 0.25 and 0.75 are used.

.(ii) median: $y=0.5$ and $y=0.92x$; so $x=0.54$; x is Brexit score and $y=f(x)$ is cumulative distribution function.

3 In the middle

3.1 Mrs May's Deal ... December 2018

Ahead of the vote ... what the papers say

December 8th, 2018.

Will phone calls to Europe win concessions? ... and will these be enough?

Will the vote be postponed? ... will the postponement be cancelled? ...

Theresa May's deal will lose Tuesday's vote. That's what everyone says. The probability is maybe 80%. Does it *deserve* to lose? The Times says 'no'. The Mail on Sunday says 'no'. The Observer says 'yes' ...

The Times, Saturday, 8 December 2018, p. 31

Least bad option. A divided parliament is about to take a historic step in severing ties with the European Union. With no enthusiasm, The Times backs Theresa May's Brexit deal.

<https://www.thetimes.co.uk/article/the-times-view-on-the-looming-brexit-vote-supporting-theresa-may-is-the-least-bad-option-b7qg3vfm3>

Mail on Sunday, 9 December 2018

Unless there is a last-minute postponement, Tuesday will be one of the most decisive moments in modern British history. All the arguments for and against **Brexit** will be considered in one place and will be resolved in our elected sovereign parliament. In theory this should mean reasonable men and women using facts and logic to reach a decision which is the best possible one for this country and its people. This is what the House of Commons is supposed to be for, a place where the real divisions of the country can be reflected, expressed and decided.

But all the signs suggest that overheated fanaticism, combined with a delusional inability to study or accept hard facts, will instead lead to a dangerous rejection of the Prime Minister's hard-bought deal, by people who offer no realistic alternative.

<https://www.dailymail.co.uk/columnists/polls/article-6475693/Mail-Sunday-comment-Tory-rebels-betraying-partys-core-beliefs.html>

The Observer, Sunday, 9 December 2018

'May's deal would be to sleepwalk into an act of national self-harm'. ...

<https://www.theguardian.com/commentisfree/2018/dec/09/the-observer-view-on-the-outcome-of-mays-brexit-deal-observer-editorial>

[As we now know, the vote was postponed.]

Analysis of a YouGov poll

YouGov

https://d25d2506sfb94s.cloudfront.net/cumulus_uploads/document/5c5i19boq1/SundayTimes_Results_191207.pdf

The contestation around Mrs May's Brexit deal is an example of a common situation. There are two opposing sides and a third agent in the middle. Under what

circumstances and according to which criteria does the middle defeat the two opposing sides?

YouGov conducted a survey on Friday 7th December 2018. Here we use the information on the tables on pages 16 and 17 of their file of data tables. These give the responses to the following questions:

Imagine there was a three-way referendum on Brexit, with the options of Britain remaining in the European Union, leaving the European Union with the proposed deal, or leaving the European Union without a deal. People would be able to vote for their first and second preference.

Which would you vote for as your first preference?

[n=1652]

Which would you vote for as your second preference?

[Only to those who gave a preference in question above; n=1439]

Notice the two ‘opposing sides’, Remain and No Deal; and the ‘middle’, May’s deal. These can be located on the Brexit Leave-Remain continuum:

Leave May’s deal Remain

The options and percentage responses are given in Table 1. Note that there were substantive replies relating to the three options by 84% and 52% of the people to the first and second preference questions, respectively. The table presents the percentages given in the report on page 16 and also recalibrated or imputed percentages, which are derived from the original percentages – the derived percentages are shown in brackets. It is the derived percentages which we shall use in the rest of the analysis.

Table 1 The first, second and third preferences; original percentages and recalibrated/imputed percentages (in brackets). All the sample.

preferences:	first, %	second, %	third, %	1+2
Remain a member of the European Union	45 (54)	4 (8)	(38)	(62)
Leave the EU with the proposed deal (May’s deal)	15 (18)	35 (67)	(15)	(85)
Leave the EU without a deal (No Deal)	24 (29)	13 (25)	(46)	(54)
.	(100%)	(100%)	(100%)	
Would not vote ... cast a second	6	42		
Don’t know	10	5		
Refused	1	1		

One of the opposing sides, Remain, has most first preferences; and, using imputed percentages, an absolute majority of first preferences; and least second preferences.

The middle, May’s deal, has least first preferences and most second preferences; and the most combined first and second preferences, and hence least third preferences. (May has most first and second preferences combined, even with the raw percentages.)

The other opposing side, No Deal, has most third preferences.

This is consistent with the notion of a middle against two opposing sides. Under what circumstances and according to which criteria does the middle defeat the two opposing sides?

Is the middle the Borda winner? Note that its poor showing on first preferences may be compensated for by its good showing on second preferences. However Remain wins with a Borda count of $2 \times 54 + 8 = 116$ versus May's deal, $2 \times 18 + 67 = 103$, and No Deal, $2 \times 29 + 25 = 83$. Note that May's deal has moved up from last to second place and is not far behind the winner.

However there is one criterion by which May's deal is the best: the middle has lowest polarisation. Polarisation can be said to occur if an option is ranked highly by many voters - but also ranked lowly by many voters. One measure of this might be the proportion of first preferences times the proportion of last preferences. According to this polarisation index, May's deal is least polarising, $0.18 \times 0.15 = 0.027$. More polarising is No Deal, $0.29 \times 0.46 = 0.113$. Most polarising is Remain, $0.54 \times 0.38 = 0.205$. So May's deal is least polarising, much less polarising than the other two options.

3.2 The VFSD model for three options

The surface situation is as follows. There are three options: Remain, or Leave with May's Deal, or Leave with No Deal. Each individual is asked which of the three they want. The surface behaviour is that each individual answers one of the three options. This gives the surface percentages, P% Remain, Q% May's Deal and $R = (100 - P - Q)\%$ No Deal. Here, $P = 54$, $Q = 18$ and $R = 28$.

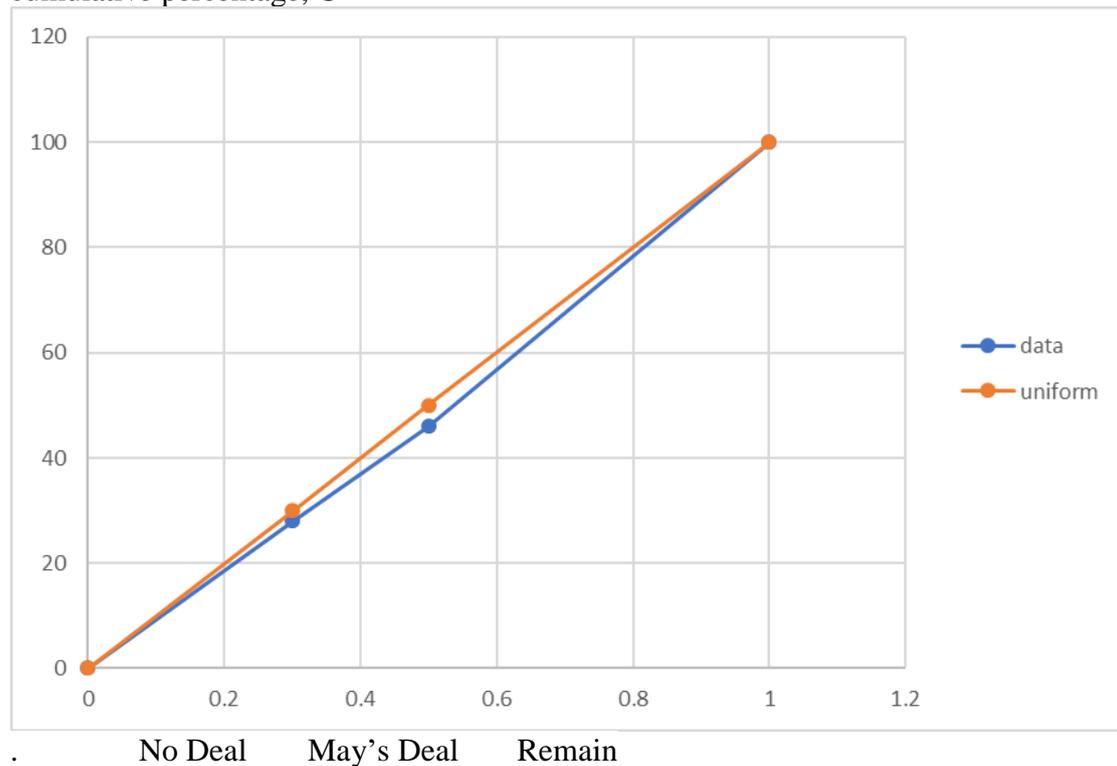
Taking this as the surface situation we now reconceptualise it in terms of the VFSD model (Value Function Spatial Distribution). Suppose that the underlying reality is that there are not three options but a continuum of options, different degrees of hard or soft Brexit. Let us represent this continuum by a number x that can take any value between 0 and 1 - between the most extreme form of Leave and the most extreme form of Remain. The borderline between Leave and Remain is at $x = 0.5$ (as before). Below 0.5 is Leave; and above 0.5 is Remain.

Where is the borderline between No Deal and May's Deal? Somewhat arbitrarily we take it to be at $x = 0.3$.

Ordered on the Brexit continuum, the options have percentages 29, 18 and 54. This gives cumulative percentages 0, 29, 47 and 100. Figure 1 below shows the resulting cumulative distribution. The shape is very close to that of the uniform distribution, perhaps with a very slight indication of a polarised distribution.

Figure 3 The cumulative distribution; data and uniform model; $C=x$

cumulative percentage, C



4 Two middles

4.1 Mrs May's Deal or a softer Brexit, March 2019

One way of challenging the Brexit impasse is to emphasise the point that there are different democratic criteria and that sometimes different criteria identify different winners.

In her recent *politics.co.uk* article, Christina Pagel and her UCL team use this approach in their analysis of You Gov data.

[https://d25d2506sfb94s.cloudfront.net/cumulus_uploads/document/vwj42ojs63/UCL_Brexit_190326_w.pdf](https://d25d2506sfb94s.cloudfront.net/cumulus_uploads/document/vwj42ojs63/UCL_Brexit_190326_w.pdf;);

<https://www.politics.co.uk/comment-analysis/2019/04/23/the-crazy-polling-of-soft-brexit>;

<https://www.thetimes.co.uk/article/it-s-not-only-mps-split-on-best-brexit-tm22qb9b3>;

The Times, 27 April 2019, p. 9; Oliver Wright “It’s not only MPs split on best Brexit.”

<https://www.politics.co.uk/comment-analysis/2019/04/15/polling-analysis-the-full-extent-of-britain-s-division-bruta>

“ it turns out that establishing what people want depends on how you assess their preferences.”

People ranked four options: Remain, softer Brexit, May’s Deal and No Deal.

“Please rank the following 4 possible final outcomes of Brexit in order of what you want to happen (1 - your favourite, 4 your least favourite).

Remain in the EU

Leave with no withdrawal agreement (No Deal)

Leave with a softer Brexit Deal (Customs Union and/or Single Market Membership)

Theresa May's Withdrawal Agreement (Deal)”

Pagel considers the following criteria: absolute majority, relative majority, most lowest ranks, one-on-one contests (Condorcet), Alternative Vote and Coombs (least disliked). Different criteria produced different results.

Christina Pagel: “not a silent majority for a compromise”.

Sir John Curtice: parliament reflects public’s impasse.

The Times, 27 April 2019, p. 9; Oliver Wright “It’s not only MPs split on best Brexit.”

politics.co.uk article, Christina Pagel and her UCL team

<https://www.politics.co.uk/comment-analysis/2019/04/23/the-crazy-polling-of-soft-brexite>;

You Gov data.

https://d25d2506sfb94s.cloudfront.net/cumulus_uploads/document/vwj42ojs63/UCL_Brexit_190326_w.pdf;

No Deal almost wins UCL’s Alternative Vote ...

No Deal at 48.1% is just behind Remain 51.9%, after elimination of other options.

Gordon: “No Deal almost wins UCL’s Alternative Vote”

Times: “the final split being 51.9% to Remain versus 48.1% for a no-deal departure”.

Christina Pagel: “leaving a one-on-one match-up between No Deal and Remain which Remain wins - but only just”.

This is just one of the findings. It is important in that some people are arguing that the Alternative Vote method should be used to decide the issue – probably not realising that it could produce a win for No Deal.

Alternative Voting – see pages 63-64 in

“Brexit and ...”

<https://docs.google.com/viewer?a=v&pid=sites&srcid=ZGVmYXVsdGRvbWFpbmxb3Jkb25idXJ0bWF0aHNvY3NjaXxneDoxNjkyNDA0YzljOWEzODY0>

May’s Deal is the second best in terms of ...

As we shall see below: Mrs May’s Deal is the second best to a Softer Brexit in terms of a few of the criteria discussed in the following section.

NOTE: in an earlier statement I had repeated an error in the paper version of *The Times* claiming more for Mrs May’s Deal. This has subsequently been corrected in the online version.

The Times, 27 April 2019, p. 9; Oliver Wright “It’s not only MPs split on best Brexit.”

Also, accessed online: 30th April 2019.

<https://www.thetimes.co.uk/article/it-s-not-only-mps-split-on-best-brexite-tm22qb9b3>;

Ten different criteria

Here I very quickly jot down the results for different criteria. This is based on the Pagel and YouGov study, sometimes with additional analysis by myself.

First preference

Remain 45.0, No Deal 27.4, May's Deal 14.1, softer Brexit 13.5

First two preferences

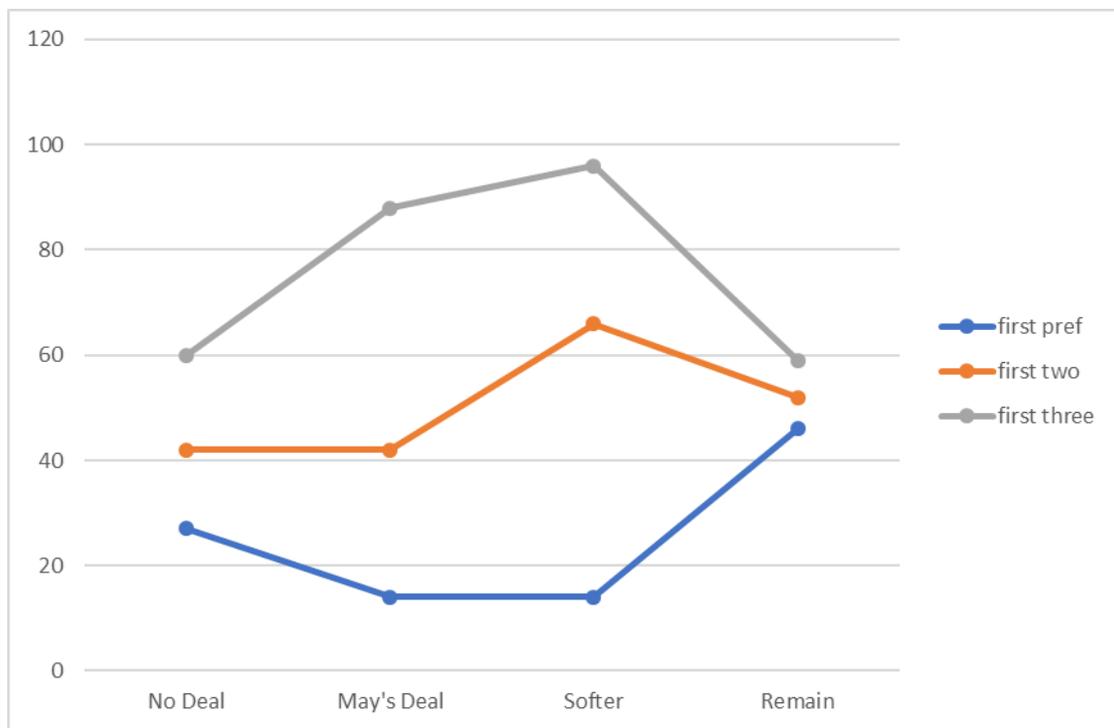
softer Brexit 66, Remain 52, No Deal 42, = May's Deal 42

First three preferences

softer Brexit 96, May's Deal 88, No Deal 60, Remain 59

The above three cumulative preferences are presented in Figure below as profiles in Brexit space. The two extremes which topped the first preferences are replaced in the first three preferences by the two middle options.

Figure 4 The cumulative preference profiles in Brexit space



The three cumulative preference profiles can be thought of as different (rather unsophisticated) weightings of the rankings. The Borda Count offers a more sophisticated (and natural) weighting.

Borda

Softer 75, Remain 56, May 44, No Deal 28

..... No Deal May Softer Remain

First (+2)	27	14	14	46
Second (+1)	15	28	52	6
Third (0)	18	46	30	7
Fourth (-1)	41	12	5	42
Total score	28	44	75	56

Condorcet

Softer beats all three, Remain, May's Deal, No Deal

AV

Remain 51.9, No Deal 48.1 in 3rd round; May Deal 20 in 2nd round; softer 13.5 in first round

Coombs, % ranking bottom

Softer 37.9 beats May 62.1 in final round (low scores are good!); No Deal 49 at 2nd round; Remain at 1st round.

Polarisation, A

Softer, least polarised ... Remain most polarised

	No Deal	May	Softer	Remain
First x fourth	1107	168	70	1932

* Median point ... on the Brexit dimension, [0,1]

Softer, in the middle because 41% to the left and 46% to the right ... median point re cumulative distribution = 0.46

* Mean point ... on the Brexit dimension, [0,1]

Between May and Softer, three quarters of the way from May to Softer, mean point = 0.425
Option points: 0.25, 0.35, 0.45, 0.55, respectively.

* Polarisation, B ... on the Brexit dimension, [0,1]

In view of Polarisation A, might least polarisation occur at a point between May's Deal and Softer?

* Polarisation, C ... on the Brexit dimension, [0,1]

Polarisation has a minimum around Softer, possibly towards Remain.

	No Deal	May	Softer	Remain
distances	180	133	114	123

(180=14+28+138; 14=14x1, 28=14x2, 138=46x3)

* These last four measures are part of what I am calling the Value Function Spatial Distribution model (VFSD).

Condorcet and Borda ... and Peter Emerson

Those of you who know Peter Emerson as well as I do will not be surprised to find that he is pressing for greater consideration of the Modified Borda Count! There is an issue here around the comparative merits of Borda and Condorcet – see:

Meta-indecision: which method should we use to decide Brexit?

pages 61-65, especially 64-65

<https://docs.google.com/viewer?a=v&pid=sites&srcid=ZGVmYXVsdGRvbWFpbmxb3Jkb25idXJ0bWF0aHNvY3NjaXxneDoxNjkyNDA0YzljOWEzODY0>

On further reflection it occurs to me that there is a distinction between Condorcet and inferred Condorcet. The YouGov survey did not explicitly ask Condorcet questions. However the survey did ask about preferences. It was from these preferences that the researchers inferred the Condorcet results. Inferred in this way Condorcet responses are always well-ordered whereas actual Condorcet responses are not always well-ordered.

Also some of the criticisms of Borda are criticisms of the preference data – so might inferred Condorcet results be vulnerable to the same criticisms?

A different point is that there is always a Borda winner whereas sometimes there is no Condorcet winner.

Finally preference data is asked about n options whereas, Condorcet data is asked about $n(n-1)/2$ pairings. Condorcet data requires more questions.

4.2 The VFSD model for four options

NOT polarised!?

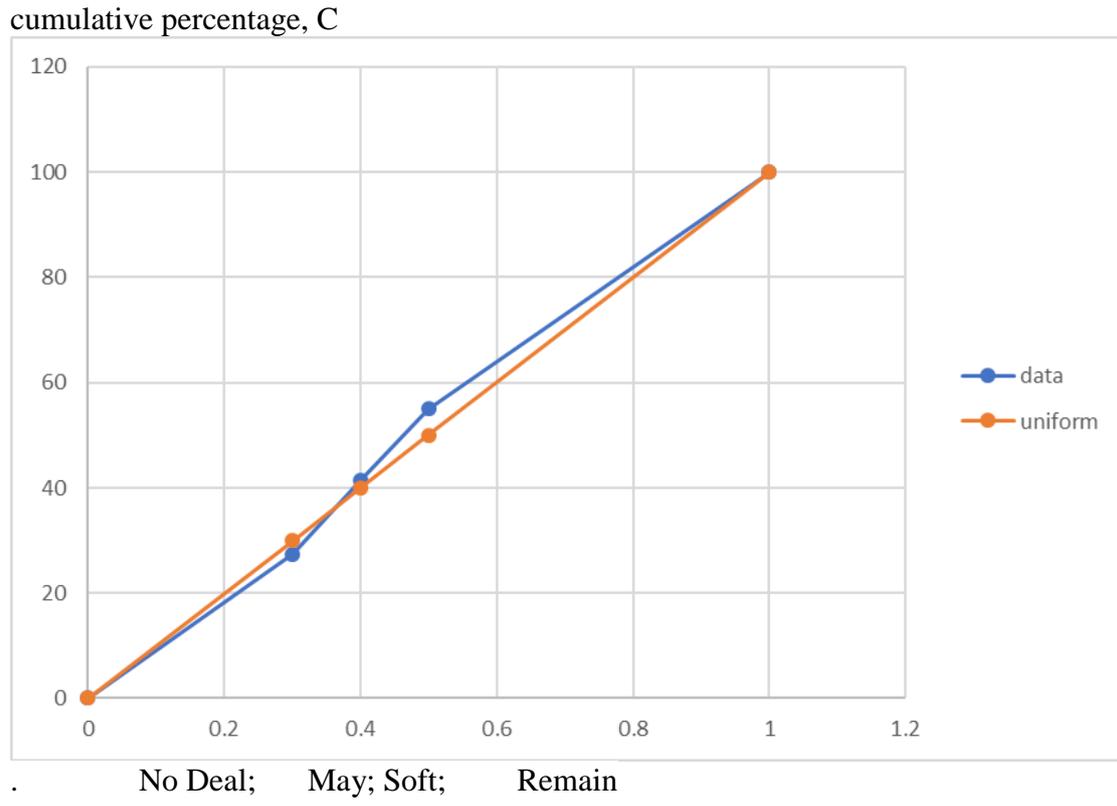
Brexit certainly feels polarised, doesn't it? Also looking at the percentages, the two extreme options have 72.4% leaving the two middle options with just 27.6%. QED. (Remain 45.0, No Deal 27.4, May's Deal 14.1, softer Brexit 13.5).

However I would like to argue that it is NOT polarised ... in the following sense. If we adopt a VFSD model then polarisation is defined in terms of the cumulative distribution over $[0,1]$ on the Brexit continuum. So let's have a look.

Ordered on the Brexit continuum, the options have percentages 27.4, 14.1, 13.5 and 45.0. This gives cumulative percentages 0, 27.4, 41.5, 55 and 100. In a previous section it was suggested that the options corresponded to points 0.25, 0.35, 0.45 and 0.55, respectively. Interval boundaries are 0.3, 0.4 and 0.5 with end points 0 and 1.

Figure 5 below shows the resulting cumulative distribution. The shape is not the shape of a polarised distribution, but rather its shape is very close to that of the uniform distribution, perhaps with a very slight peak around $x=0.4$.

Figure 5 The cumulative distribution on the Brexit dimension; data (the respondents in the Pagel study) and uniform model; $C=x$



Single-peaked preference functions in Brexit space

An important result in voting theory is the median voter theorem. It may be possible to order options along a continuum in such a way that each voter’s preferences are single-peaked. If this can be done then the Condorcet winner corresponds to the option most preferred by the median voter.

In the Brexit situation, a meaningful ordering is: No Deal, May, Softer and Remain. With this ordering the median voter votes Softer. Also the Condorcet winner is Softer. So this is consistent with the median voter theorem. But it still leaves open the question as to whether individuals’ preference orderings are single-peaked. Let us check this out.

First consider the following table in the YouGov report.

	No Deal	May	Softer	Remain
First (+2)	27	14	14	46
Second (+1)	15	28	52	6
Third (0)	18	46	30	7
Fourth (-1)	41	12	5	42

Note the dominance of the two main diagonals – this looks like the strict ordering of the preferences for No Deal voters and Remain voters. This is sufficient to give an initial impression of single-peakedness at a group level. See figure 6.

5 Locating the parties in two-dimensional political space

The previous sections have all involved just the one dimension. In this section we take the first step into two dimensions.

The underlying reality is that people, parties and policies are located in political space. An awareness of this space is indicated in everyday discussion with references to ‘Left’ and ‘Right’, and to ‘hard’ and soft’ Brexits. We now locate parties in a two-dimensional political space, consisting of the familiar left-right dimension and also a second dimension: the Brexit dimension.

The ordering of the parties on the Left-Right dimension is indicated below. The justification of the ordering here comes from the gravity model analysis of the 2017 election.²

The Left-Right dimension:

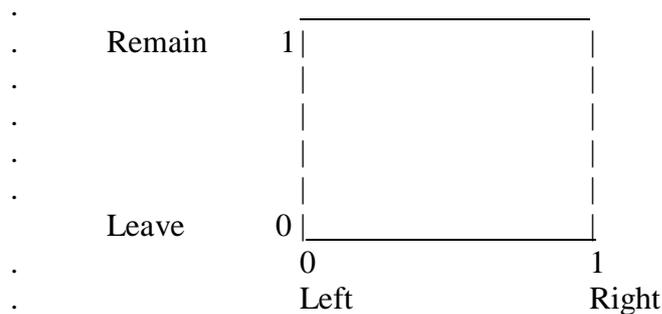
SNP/PC, Labour, Green, Change, LibDem, Conservative, Brexit, UKIP.

However there is another dimension by which we can order the parties, namely the Brexit dimension. Here Labour and Conservative are in the middle, sandwiched between UKIP-related parties and Remainer parties. The ordering of the parties on the Brexit leave-Remain dimension is indicated below. The justification of the ordering here comes from the indicative voting by MPs.

The Hard-Soft Brexit dimension:

UKIP, Brexit; Conservative, Labour; SNP/PC, Green, Change, LibDem

We now suppose that scores on these two dimensions run from 0 to 1.



Parties as points

First consider the Left-Right dimension. Suppose we put the parties into five party groupings, in the following way:³

SNP/PC; Labour; (Green, Change, LibDem); Conservatives; (Brexit, UKIP)

Suppose we assign scores to the party groupings:

0, 0.25, 0.5, 0.75 and 1

² Burt, Gordon. *Values, World Society and Modelling Yearbook 2017*. Newcastle: Cambridge Scholars. pp. 78-80.

³ (Sometime in mental and hand calculation the scores of -2, -1, 0, 1 and 2 are easier to work with, and can at the final stage be transformed.)

Next consider the Leave-Remain Brexit dimension. Suppose we put the parties into four party groupings, in the following way:⁴

UKIP etc; Conservatives; Labour; Europhile (SNP/PC, Green, Change, LibDem);

Suppose we assign scores to the party groupings:

0, 1/3, 2/3 and 1

All this conceives of parties as points ...

Parties as intervals

... however people in the same party have different views and so it is more appropriate to represent a party as occupying an interval along a dimension.

Five intervals on the Left-Right dimension:

[0,0.2] [0.2,0.4] [0.4,0.6] [0.6,0.8] [0.8,1.0]

Four intervals on the Brexit Leave-Remain dimension:

[0,0.25] [0.25,0.5] [0.5,0.75] [0.75,1.0]

Parties as mid-points of intervals

Sometimes it is useful to think of parties being at the mid-points of their intervals. We shall do this in our calculations in later sections.

Five mid-points of intervals on the Left-Right dimension:

0.1, 0.3, 0.5, 0.7, 0.9
[0,0.2] [0.2,0.4] [0.4,0.6] [0.6,0.8] [0.8,1.0]

Four midpoints of intervals on the Brexit Leave-Remain dimension:

0.125, 0.375, 0.625, 0.875
[0,0.25] [0.25,0.5] [0.5,0.75] [0.75,1.0]

Each party grouping has a mid-point on each of the two dimensions, giving a point in two dimensions – see Figure 7.

⁴ (Sometime in mental and hand calculation the scores of -3, -1, 1 and 3 are easier to work with, and can at the final stage be transformed.)

Figure 7 Five party groupings in two-dimensional political space (unitised)

